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The first experimental works which pertain to this problem were conducted by V. V. Baturin and I. A. Shepelev [16], Yu. V. Ivanov [17] and G. S. Shandurov [1]; as a result of these works empirical formulas were recommended for the calculation of the form of the axis of the jet which turned out to be dependent on the relation of dynamic pressures in the initial jet cross-section and incident flow and on the initial angle of slope of the jet.

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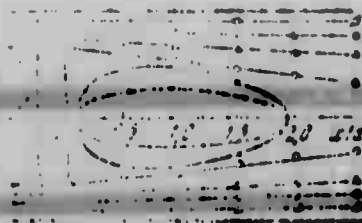


Fig. 14.

Fig. 14. Flow lines in a jet of low velocity which flows out into a flow of high velocity ($m = 27$).



Fig. 15.

Fig. 15. Flow lines in a liquid being sucked in toward a flat submerged jet which flows out from an opening in the wall.

Subsequently, M. V. Volynskiy and I [1] developed approximate methods for the calculation of the form of the axis of the jet which are confirmed by the data of the indicated experimental works. Similar studies of other authors are also known. Recently two works of T. A. Girshovich [18, 19] have been published in which this problem for a plane jet is solved by a more rigid theoretical method, whereupon it is possible to find not only the form of the axis of the jet but also its boundaries and velocity profiles in different transverse cross sections, i.e., to construct the entire flow as a whole.

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The problem is solved in a curvilinear coordinate system, the axis of which coincided with the axis of the jet, and the ordinate is orthogonal to the axis of jet. The boundary layer equations are written in this coordinate system for the zone of mixing taking into account the pressure field being created by centrifugal forces and the variable accompanying velocity. For determining the external jet boundary (from the direction of the incident flow) the latter is considered conditionally as the boundary surface of the current obtained from the addition of the incident irrotational flow with the system of the sources arranged on a line parallel to the incident flow and passing through the origin of the jet (Fig. 19), whereupon the distribution of the sources is selected from a supplementary condition which is related to the fact that the pressures on the jet boundary and the incident flow are identical.

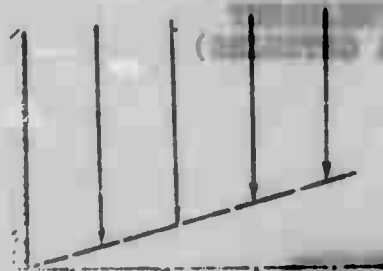


Fig. 16.

Fig. 16. Flow lines in a liquid being sucked in toward an axisymmetric submerged jet which flows out from a slot in a wall.

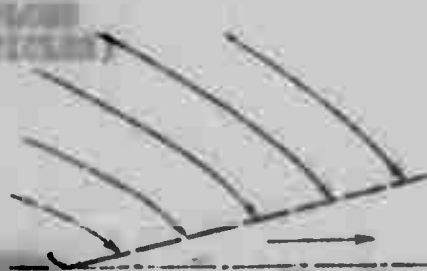


Fig. 17.

Fig. 17. Flow lines in a liquid being sucked in toward an axially symmetrical jet which flows out from a nozzle.

The axis of the jet calculated by T. A. Girshovich and its boundary are plotted on Fig. 19 and the corresponding experimental points are given; furthermore, the velocity fields in two orthogonal jet cross-sections are depicted (the experiment was conducted with a plane jet of air which flows out at an angle of 90° to the airflow with ratio of velocities $u_H/u_0 = 0.2$).



Fig. 18.



Fig. 19.

Fig. 18. Flow lines in a liquid being sucked in toward an axially symmetrical jet which flows out from a tube which is inserted flush into a plate (the jet flows out on a flat screen).

Fig. 19. Configurations of an airflow which flows out into a lateral airflow at an initial angle of 90° with ratio of velocities $u_H/u_0 = 0.2$ (points - experiment).

Figure 20 gives the theoretical curve and the experimental points of change in velocity along the axis of a jet for the same conditions. G. S. Shandurov indicated a simple method of considering the dissimilarity of the densities of lateral flow and a jet consisting in the fact that with equal values of the relations of dynamic pressures the pictures of the flow coincide.

In certain cases it is necessary to deal with turbulent jets subjected to the action of gravitational forces. If the direction of a jets of gas which has in the initial cross section a density which differs from that in the environment differs from the vertical, the gravity distorts it. In the works of S. N. Syrkin and of D. N. Lyakhovsky [20] the forms of the axis of a jet of heated air which flows out into air of normal temperature are experimentally investigated; the axis turned out to be more distorted the greater the preheating of the air. V. V. Baturin and I. A. Shepelev [21] and G. N. Abramovich [1] developed theoretical methods of calculation of the form of a distorted jet. It turned out that all the

experimental data can be placed on a single theoretical curve if we introduce into the calculation Archimedes's criterion

$$Ar = \frac{u^2 \Delta T_0}{g d \frac{T_H}{T_0}}$$

(G. A. Abramovich's comparison of experimental points with the calculated curve is given on Fig. 21).

An interesting example of the use of curvilinear nonisothermal jets, called fountains, is the ventilation of the large exhibition pavillion in Sokolniki Park (Moscow). Jets of cold (street) air are fed to the premises from several inclined slotted openings arranged along opposite walls of the pavillion (Fig. 22). Under the action of the initial impulse these jets rise upward, but the air density in cold jets is greater than in the air of the pavillion, in consequence of which the vertical velocity in the jets gradually decreases; at some height the initial impulse is balanced by the Archimedes force directed downward, whereupon the jet, under the action of the latter, begins to drop and finally it comes into the operating zone of the pavillion but, in this case, the jet already manages to warm up - and thus in the working zone where the visitors of the pavillion are located it is possible to create comfortable conditions. The described ventilation system was designed according to I. A. Shepelev's theory.

In recent years theoretical and experimental studies have been conducted of the convective jets which arise near heated horizontal [1] jet of gas [22] surfaces.

There are important results for the turbulent jets in which Lyakhovskiy (flames of combustion) occurs; without dwelling on this question, we will refer the reader to the appropriate literature [1, 23].

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The Physics of the Jet of a Turbine

Diagram illustrating the change in velocity of a jet

Calculation of the change in velocity of a jet

Calculation of the change in velocity of a jet

Calculation of the change in velocity of a jet

Pressure fluctuations in the boundary of a turbulent flow

Turbulent flows in jets and wakes

The turbulent mixing of two fluids

Experimental study of the mixing of two fluids

On the mixing of two fluids

Diffusion and vortex points of a turbulent flow

Diagram of an underexpanded supersonic jet

Diagram illustrating the dependence of the coefficient of average velocity

Diagram illustrating the dependence of the coefficient of average velocity

Diagram illustrating the dependence of the coefficient of average velocity

Diagram illustrating the dependence of the coefficient of average velocity

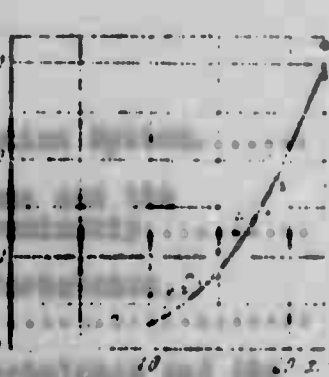


Fig. 20.

Fig. 21.

Fig. 20. Change in dimensionless velocity along the axis of a jet which flows out into a lateral airflow at an initial angle of 90° with ratio of velocities $u_0/u_0 = 0.2$ (points - experiment).

Fig. 21. The axis of a submerged air jet distorted by gravitational forces (points - experiment).

Fig. 22. Fountains of cold air (the axis of the fountain is shown by the dot-dash line).

Fig. 23. Diagram of an underexpanded supersonic jet and curves of the dependence of the coefficient of average velocity ($\lambda = u/a_{np}$) on the relative area of cross section ($f = F/F_a$) calculated from conditions for the conservation of momentum J and the weight-to-mass flow rate of gas G (in the initial and maximum cross sections both conditions are satisfied simultaneously).

An interesting study of an off-design supersonic jet was performed by B. A. Zhestkov, M. M. Maximov et al [1]. A simple method for the determination of the form of the jet on the gas-dynamic section was proposed by A. Ya. Cherkez [24]; a detailed experimental and theoretical investigation of the isobarometric section of a supersonic jet was conducted by Chiang Tse-hsing [27]. In A. Ya. Cherkez's mentioned work, the calculation of the gas-dynamic section whose diagram is given in Fig. 23 is conducted by the methods of one-dimensional gas dynamics with the use of equations of conservation and without consideration of the mixing-in of the surrounding environment with the jet.

An important result of the work of B. A. Zhestkov, M. M. Maximov et al is the establishment of the fact that the damping of velocity on a large part of the isobarometric section of the jet is expressed in logarithmic coordinates by parallel straight lines that have the same slope as in the jet of a noncompressible liquid; the effect of the degree of off-design and the initial value of the Mach number is manifested primarily in the shifting of the point where the drop in velocity on the axis of the jet begins, i.e., in a change in the abscissa x_n of the beginning of the main section of the jet (Figs. 24 and 25).



Fig. 24.



Fig. 25.

Fig. 24. Curves of the change in dimensionless velocity along the axis of a supersonic jet with different values of the degree of off-design $n = p_a/p_H$ for $M_a = 1.5$.

Fig. 25. Curves of the change in dimensionless velocity along the axis of a supersonic jet with different values of the degree of off-design $n = p_a/p_H$ for $M_a = 3$.

Everything said pertains to a submerged supersonic jet. With the presence of a cocurrent flow, the picture of the flow becomes complicated and up to now has been little studied.

Work in the field of turbulent gas jets is being conducted at the present time in many domestic and foreign laboratories. One of the important directions of these studies is research on the turbulent microstructure of a jet and the establishment of the direct connection between it and the averaged flow conditions. Only the first steps in this direction thus far have been made in the works of A. S. Ginevskiy et al [25], G. N. Abramovich et al [10] and some studies of foreign authors [26].

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THE PROBLEM OF THE STABILITY OF LAMINAR FLOW AND THE TRANSITION TO TURBULENCE

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Summary

At the present time the theory of aerodynamic stability has been converted into a large independent section of science covering a very wide circle of interesting scientific and engineering problems.

Pertaining to this section are the classical problems of flow stability in tubes, ducts, and wing boundary layer, the problem of the flow stability of a viscous liquid in limited volumes, and problems of the stability of wind in the atmosphere and of the stability of heat convection. Pertinent here are interesting problems of the flow stability of the conducting liquid in a magnetic field, the problems of the flow stability of plasmas, and many others.

The circle of these questions is so broad that it cannot be illuminated in one report. Thus, we will dwell only on some questions of hydrodynamic stability which have the greatest relationship to the phenomena of the transition of the laminar flow

to turbulence and to the problem of the laminarization of the boundary layer.

These phenomena are exhibited most distinctly in flows of a viscous fluid in an unlimited area - in the fluid flows in tubes, ducts and in the wing boundary layer. We will also dwell on these problems because, on the one hand, until now they are insufficiently developed and, on the other, because at the present time they acquired great scientific and practical value.

THE TURBULENT MIXING OF FREE GAS JETS

In contrast to these problems, in the study of the stability of the viscous flow in limited volumes, including studies in nonlinear stability theory, in recent years considerable progress has been achieved. A whole series of very interesting and mathematically rigorous conclusions have been obtained here, and in particular the applicability of the method of small oscillations has been demonstrated.

1. Very considerable attention is given in domestic and

foreign literature to the study of the turbulent jets of liquid and gas in recent decades. This is explained first of all by the wealth and diversity of engineering applications of the jet flows. At the same time, their examination makes it possible to obtain valuable information about the mechanism of turbulent motion and turbulent transfer. In this respect it is significant that after a temporary decline, interest increased noticeably again in the detailed study of the pulsating structure of turbulent flows, in particular jet flows. Especially promising for the present are isolated attempts at active influence (mechanical, acoustic, electromagnetic - for a conducting medium and others [1-3]) on the development of jets.

If, at the first stage, during the creation of the theory of turbulent jets, the main thing was the development of general regularities (similarity of flow and, in particular, the self-similarity of jets according to Reynolds number), then at the present time the primary thing becomes the clarification of the finer features of the flow and the role of different, in the first

approximation "secondary" factors. In light of the aforesaid, let us examine some general questions of the theory of turbulent jets and, in more detail, the results of one of the attempts at function the active influence of a flow [1, 11].

2. The theory and the methods of calculation of turbulent jets are one of the developed sections of the contemporary semi-empirical theory of turbulence. Without going into details, let us note the satisfactory solution, in general, of the basic problem of such theory for a number of jet flows.) Drawing on limited empirical information, it is possible to arrive at sufficient agreement (frequently within the limits of the accuracy of experiment) of the calculated and experimental data for the middle flow (in Reynolds' sense). As a rule, this can be achieved in different ways. Actually, different semi-empirical methods of calculation of turbulent jets (based on the scheme of an asymptotic boundary layer or layer of finite thickness, on the integration of differential equations under specific assumptions about "turbulent viscosity" or on the use of integral relationships, on the polynomial representation of the velocity profile or friction stress or on the a priori selection of the profile, etc. [4, 5, 6]) in a number of cases, especially for self-similar jets, lead to a satisfactory description of the middle flow. It is more complex with non-self-similar jet flows; however, even here different interpolation schemes ([4, 6, 7, 8] and others) and especially the method of the equivalent problem of the theory of thermal conductivity [5] prove to be sufficiently effective for engineering calculation in certain cases.

The fundamental problems standing in this area are connected with the expansion of a nevertheless comparatively narrow circle of flows which yield to calculation and to the reduction, to the minimum, of the data borrowed from experience. As concerns the selection of a more effective method of calculation, (for turbulent jets of a noncompressible liquid) this question is not so fundamental.

a) One should, however, indicate certain advantages of the method of

the small parameter $1/\text{Pr}$ which considers the viscosity effect; an equation not allowing for viscosity has altogether only a second order;

the asymptotic layer over the layer of the finite thickness and the necessity for a difference in (the nominal thicknesses of) the dynamic and thermal layers (in accordance with the value of the "turbulent Prandtl number" on the order of 0.7-0.8 for the thermal and diffusion problems) and others.

Many first attempts at the solution of equation (6) by approximation also waiting its turn is the expansion of the investigation [2] of the balance of fluctuating energy for problems of the theory of turbulent jets for which this method, apparently, will be very large. The results can become unstable. This conclusion did not correspond to the point of view which existed then and it was not with great 3. More complex than for the noncompressible liquid is the matter of the study and calculation of the turbulent jets of a compressible gas. Characteristic of the latter is a certain contradiction in experimental reference data on turbulent mixing. While some authors [4, 9] consider the intensity of turbulent mixing in cocurrent gas jets minimum with the identical values of velocity in the jet and surrounding environment [4, 9], others cite data which testify to the presence, in the conditions of the experiment, of a minimum in the intensity of mixing with approximately identical values of u^2 [5, 10]. Finally, assertions are also encountered about the determining role of the value u^2 . However the question as a whole is much more complex. 420.

Figure 1. Each of these conclusions is based on experimental data obtained under specific conditions. Therefore, it is difficult to assume that the divergence is caused only by the difference in the procedure for processing the experiment alone (in some works the judgement on the intensity of the mixing is based on a comparison of secondary characteristics - the thickness of the mixing region [9], in others - on the direct mixing of the gas from the jet and the cocurrent flow [10]). It is more probable that under the different conditions of

¹For the outflow of a jet of denser gas into the cocurrent flow of a less dense gas.

small the experiment, the effect of the initial conditions difficult to consider is felt - the velocity profile, the initial level and scale of turbulence, intermittance, and also the value of the Reynolds number. The role of these factors is also insufficiently clarified for the turbulent jets of a noncompressible liquid. It is known, in particular, that with a decrease in number $Re = u_0 d / \nu$ (where u_0 is the exhaust velocity) the intensity of attenuation of a submerged turbulent jet does not fall, which would be natural at first glance, but increases noticeably. Apparently, the mutual implication of molecular and molar effects in the initial and transitory sections of the jet has a substantial effect.

2. With a somewhat increased (to 8-10% and more) initial turbulence, as tests show [5], a unique developed regime of the turbulent flow is accomplished. In this case the profiles of values of pu^2 , etc., in gas jets (and the burning flame [10]) become universal. Under these conditions the leading role of the difference (in values of pu^2 in turbulent mixing (in the range of values of parameters studied in the experiments) is usually exhibited. On the other hand, with the effective suppression of the initial turbulence (for example, with outflow through extremely fine grids which rarely lower the value of the scale of turbulence) the regular laws of molecular (and close to it fine-scale) mixing are more strongly pronounced, especially in the initial section of the jet. These representations need direct experimental check. And although specially posed experiments with the diffusion combustion of a gas it is showed that the length of flame is noticeably greater (and, consequently, mixing is worse) other conditions being equal if the values of pu^2 in the jet and cocurrent flow are identical [10]), here too the effect of secondary factors could be felt.

(7)

4. For an investigation of the features of the development of a turbulent jet under conditions of an increased (in which regard controlled) initial level of turbulence, experiments were conducted

With the participation of K. Ye. Dzhaygashtin and I. A. Kel'manson.

In individual particular cases the boundary-value problem in equation (A) for given α and β will have an infinite number of solutions on a unit with a mechanical vortex generator described in the work. The preliminary results of the examination make it possible to draw a number of conclusions apropos the structure of the middle and fluctuating flows in such a jet. It was made clear that for a field of average velocity the determining criterion is the value of the Strouhal number $Sh = nd_0/u_0$ (where n is the number of revolutions of the disc of the vortex generator, d_0 - the diameter of the nozzle, u_0 - the exhaust velocity). With an increase in the Sh number the intensity of turbulent mixing increases noticeably. Some data which illustrate this are given in Fig. 1 (change of velocity on the axis of the jet at different values of Sh number) and Figs. 2 and 3 (velocity profile - Fig. 2 - and temperature profile - Fig. 3 - in the transverse cross sections of a weakly heated jet). Analogous data were obtained with a considerable variation in the conditions of the experiment (with $d = 10, 20$ and 40 mm; $u_0 = 20-130$ m/s; $n = 0-250$ r/s).

For the characteristic of the total intensifying action of the vortex generator let us note that the air flow rate in the jet was subordinated to a relation of the type (for cross sections with ratio $x/d \leq 20$) $m/m_0 = 1 + 0.2 k (Sh) x/d$, where $k = 1; 1.18; 1.36$ and 1.44 respectively for numbers $Sh \cdot 10^2 = 0; 3.7; 8.0$ and 10.5 . In this, in all cases number $Pr_{turb} \approx 0.75$.

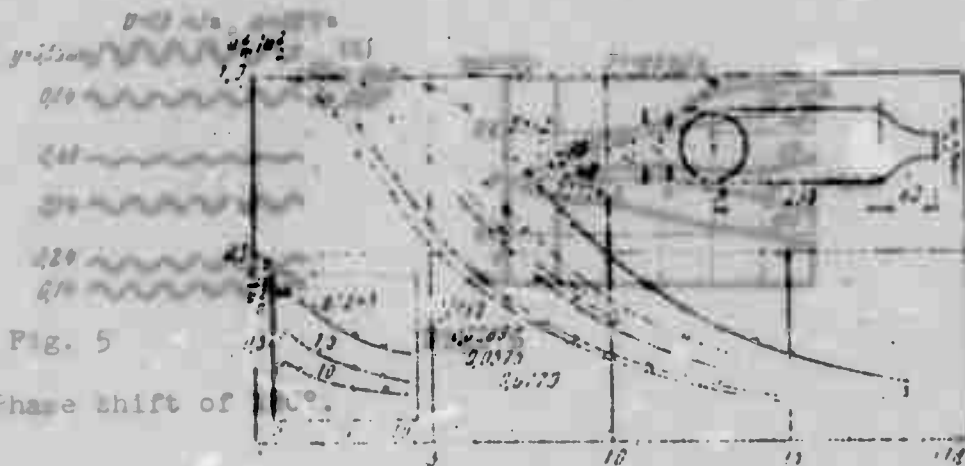
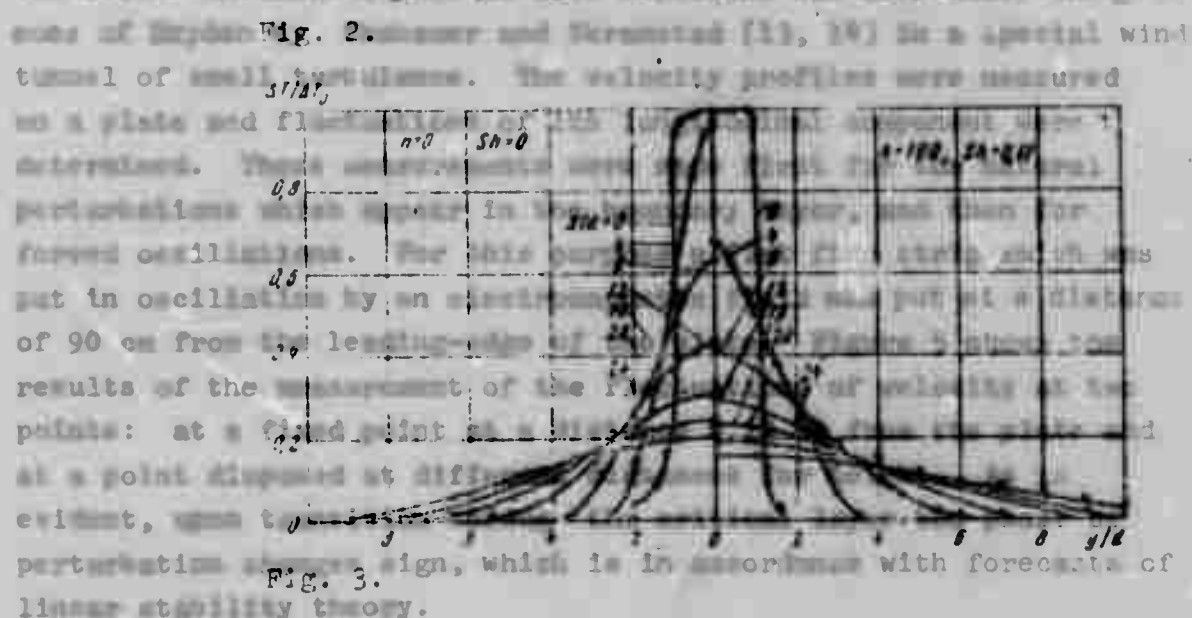
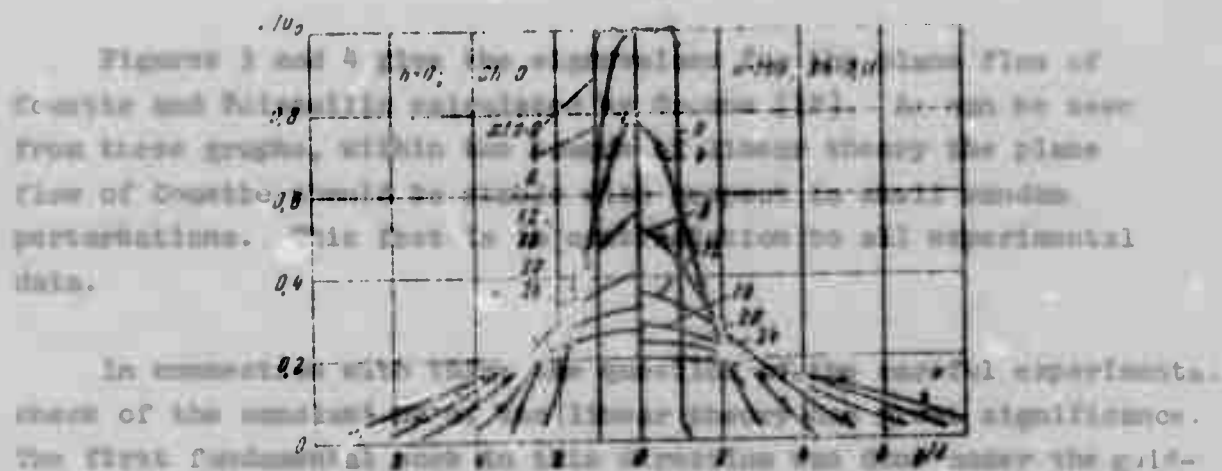


Fig. 1.



As concerns research on the fluctuating structure of a jet (still not completed), interesting conclusions were obtained with visual and photographic observations of the fluctuations of velocity with the aid of an electrothermoanemometer. It became clear, first of all, that in the investigated jet with low-frequency fluctuations superimposed with the aid of the vortex generator in general one should distinguish three characteristic regions of flow. In the first of them, adjacent to the mouth of the nozzle, the flow is quasi-regular; the oscillation frequency is equal to double the number of revolutions of the disc. In the second - transitory - developing turbulent fluctuations are superimposed on the basic

forced oscillations. Finally, in the third the characteristic irregular turbulent spectrum of fluctuations is observed. As for concerns the dimensions of the regions occupied by each type of flow, they depend on the value of Sh number. With Sh number ≥ 0.1 - ≈ 0.12 practically the entire jet is a region of developed turbulent motion; whereas with small values of Sh number there is a distinctly expressed section of a "fluctuating" quasi-monochromatic jet in it. The qualitative picture of the flow in a jet with Sh number ≈ 0.005 is presented on Fig. 4; where the boundaries of characteristic zones and also the profiles of average speed are shown. The same figure presents the characteristic oscillograms of fluctuations for in all three zones (and for comparison with number Sh = 0).

As concerns the value of the intensity of fluctuations, with a sufficiently large Sh number it is higher than in a usual jet. Thus, with number Sh = 0 at the nozzle edge $\xi_u = \bar{u}'/u_0' = 0.5-1.0\%$, and at the stream with $x/d = 10$ on the axis of jet $\xi_u = 18\%$. With number Sh ≥ 0.03 the initial value $\xi_u = 10-12\%$, and the maximum (on the axis) is shifted to $x/d = 4$ and is equal to $\xi_u = 32\%$.

As can be seen, most complex is the nature of the change of value ξ_u (and also data with the entire picture of the flow) in the first region of the flow - cases, with quasi-regular fluctuations, for the interpretation of which additional measurements will be required (specifically, frequency characteristics). A considerable difference in critical Reynolds numbers was obtained:

The obtained data as a whole make it possible to assume that with the further examination of a jet with forced fluctuations, along with the obvious applied results, additional information can be obtained about the structure of a free turbulent flow.

Specifically, this pertains to the picture of the transition from the initial, approximately monochromatic, low-frequency fluctuations, typical given by the vortex generator to the complex turbulent spectrum. With a small degree of turbulence of the incident flow cannot appear immediately. Some segment of the length of the time interval



is necessary for the... difference... in critical Reynolds number... two orders - this is already... The transition region... by... actually... entire... fluctuations... are practically... smooth, with the exception of the terminal... region...

As follows, the... of... of the experimental... of... of... in... present... to... with... there... is... of... of... conducted... turbulence...

It... revealed... generally... of... research... the... transition... has... In... every... In... a... of... In... in... in... of... of... during their movement...



Fig. 9

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Fig. 10



Fig. 11

Thus, turbulent spots and turbulent plugs are an important phenomenon for understanding the transition process. In the experiments of Dussauer and Strumelj with a plate (with a small degree of disturbance of flow) and in the experiments of Reiss in a tube with small Reynolds numbers (i.e., with slight perturbations of the initial section of a tube) the length of the transitional region turned out to be very large. In a large part of this case, the flow fluctuations are practically absent and turbulent spots

appear actually at the end of this zone. The question arises why, on such a large section, is the increase in perturbations not realized in accordance with the exponential forecast by the linear stability theory? It seems to us that in this case (supercritical) the linear theory is no longer valid in x^* . In this case the task of the development of the aerodynamic disturbances should be solved taking into account nonlinear terms in the equation of motion.

EXPERIMENTAL STUDY OF THE MIXING OF ISOTHERMAL AND NONISOTHERMAL TURBULENT COCURRENT JETS IN NOZZLES

It is known that the equation of motion leads to the appearance of new harmonics, to the interaction of different turbulent oscillations, and to the complication of the flow pattern. After the point of the loss of stability nonlinear laws of development of the aerodynamic perturbations begin to act which gradually complicate the flow pattern in the transition region and lead to the generation of turbulent spots and

The study of turbulence characteristics in flows in ducts of variable cross section is an important problem of the aerodynamics of viscous flow. Sufficiently complete information about the turbulent parameters over a wide range of flow conditions is necessary for the creation of methods for the calculation of flows in ducts which rest on the real representations of the process and use physical parameters for the description.

Such a parameter which describes diffusion in the development of aerodynamic disturbances in the direction of the flow is the mean square deviation of liquid particles (dispersion) from the middle lines of current. The use of dispersion averaged over cross section forms the basis of the diffusion model of a free turbulent jet [1].

The use of dispersion for the description of the mixing process in ducts of variable cross section is complicated by the fact that unlike the case of the free jets its change is determined not only by the processes of mixing, but also by the deformation of the flow with a change in the area of the cross section.

Let us examine the picture of the flow in the unit which was used in the experiments (Fig. 1). A heated jet which consists of the combustion products of gasoline rarefied by air was partially mixed with a cocurrent flow of cold air and it fell into a narrowing nozzle. Experiments were carried out with the identical velocities of the jet and the cocurrent flow. With the flow of a nonisothermal flow in the nozzle the shift of longitudinal velocities is developed. This is connected with the fact that, as is established by experiment, the total pressure on the nozzle edge is constant, i.e., in the process of flow in the nozzle a transition proceeds from a flow with constant velocity to a flow with constant velocity coefficients in the cross sections. $M > 1$.

The characteristic determined by the mixing process is the difference in the dispersion of a real flow and a hypothetical flow under the assumption of the absence of mixing on the investigated section of the nozzle. In this case, as will be shown below, for the description of the process in the nozzles it is more convenient to use dispersion $\sum_c^2(x) = [r_k^2/r_c^2(x)]\sigma_c^2(x)$, reduced to the radius of a cylindrical chamber, and to characterize the mixing by the difference $\sum^2(x) = \sum_c^2(x) - \sum_{co}^2(x)$.

5. The expression for determining the change $\sum_{co}^2(x)$ in the absence of mixing in the nozzle can be obtained from the condition for the retention of surplus enthalpy. Taking the form of the profile of the stagnation temperature in the form of Gaussian curves (i.e., considering real profiles in the entrance mouth of the nozzle sufficiently broad and the diameter of the chamber so large that an increase in temperature at the walls due to mixing is insignificant), the extreme value of dispersion \sum_{co}^2 attained in cross sections with constant velocity coefficients can be found from the expression (with constant velocity in the initial cross section of the nozzle)

which is obtained from the same expression of linear theory as for the case of a constant velocity in the initial cross section of the nozzle. The expression for \sum_{co}^2 is obtained from the energy equation

where σ_H^2 is the dispersion at the end of the cylindrical chamber and
 T_1, T_2 - the maximum and minimum stagnation temperature in the
 air entrance mouth of the nozzle.



Fig. 1. Experimental unit: 1 - precombustion chamber; 2 - differential thermocouples; 3 - electric air traversing equipment; 4 - equalizing grid (distance between centers of holes 9 mm), diameter of holes 6 mm); 5 - asbestos heat insulation; 6 - movable thermocouple for the measurement of temperature of jet.

KEY: (1) Air.

Constant γ was determined by charts according to the linear theory. In the region of transition from a flow with constant velocity to a flow with the constant velocity coefficients, the value $\sum_{co}^2(x)$ is described by an approximate expression valid with small distances from the entrance mouth of the nozzle:

$$\sum_{co}^2(x) = 3 \ln \frac{T_1}{T_2} \left[\ln \frac{T_1}{T_2} \frac{1 - \lambda_c(x)}{1 - \lambda_c(0)} + \left(\frac{T_1}{T_2} - 1 \right) \frac{1 - \lambda_c(x)}{1 - \lambda_c(0)} \right] \pi(x) \quad (2)$$

where $\pi(x) = \left[1 - \frac{\kappa - 1}{\kappa + 1} \lambda^2(x) \right]^{\kappa/(\kappa-1)}$, λ is the velocity coefficient in the nozzle with isothermal flow. In the case of a constant velocity coefficient at the end of the cylindrical chamber or with an isothermal one-dimensional flow $\sum_{co}^2(x) = \sigma_H^2$. (17)

A very In the extreme case of small overheatings, when the longitudinal velocity component in each section is constant, the mixing is determined by the behavior of the coefficient of turbulent diffusion along the nozzle, whereupon under the assumption of the equidistance of all averaged lines of flow (in dimensionless coordinates)

$r_x/r_c(x) = \text{const}$ parameter $\sum^2(x)$ is connected with the value of the transverse diffusion coefficient by the relationship

In this case, when $x_1 = 0$ the initial system of equations can be approximately reduced to a system of two equations for \sum^2 : (3)

The solution of this system is sought in the form where u is longitudinal velocity. An analogous relationship for dispersion σ^2 is more cumbersome and takes the following form [2]:

$$\frac{d\sum^2}{dx} + \frac{2}{r_c} \frac{dr_c}{dx} \sum^2 = 2 \frac{D}{u} \quad (3)$$

The measurements of the quoted dispersion $\sum^2(x)$ with small overheatings permit, in accordance with relationship (3), investigating the behavior of the coefficient of turbulent diffusion with deformation of the flow. Figure 2 shows experimental values of $\sum^2(x)$ with the mixing of the weakly heated jet in a Vitashinskiy nozzle (length 400 mm, diameters 200 mm and 85 mm, pressure gradient critical) and the results of the calculations (curves 1 and 2).

The available results of the theoretical investigation of the rapid deformation of turbulence obtained in [3] make it possible to determine a change in the longitudinal and transverse mean-square velocity component along the nozzle.

V. M. Iyevlev obtained formulas for the change in the scales of turbulence with rapid axisymmetric deformation, which made it possible to obtain relationships for the diffusion coefficient. Apart from this, V. M. Iyevlev examined the case of the gradual deformation with which the isotropy of turbulence manages to be established.

The calculation of the transverse diffusion coefficient under the assumption of rapid deformation shows its increase in the first quarter of the length of the nozzle up to the limiting value of $\sqrt{1/3}$ times, and with gradual deformation the coefficient of turbulent diffusion does not change. Function $\sum^2(x)$ for rapid (curve 1) and gradual (curve 2) deformations was found from relationship (3).

Used in the calculations was the experimental value of the coefficient

of turbulent diffusion in the chamber which was found by means of the measurements of profiles in the cylindrical compartment established between the nozzle and the chamber (a straight line for approximating the experiments is shown by the dotted line on Fig. 2).

In the stable region of the flow all subsequent

The measurements of the temperature profiles were conducted with differential chromel-copel thermocouples, the movable joints of which were raised simultaneously by electric air transversing equipment and could freely emerge from the nozzle if necessary. In the analysis of the experiments, the surplus stagnation temperature was taken as the parameter which coincides with the concentration.

It is possible to show that if the change in the quoted

dispersion $\sum^2(x)$ in the isothermal flow is equal in the nozzle and the cylinder, then in both cases mixing occurs equally, i.e., a drop in the axial concentration of the jet (or the maximum excess overheating) occurs equally and the profiles of concentration coincide with an affine increase in the cross section of the nozzle up to the dimensions of the cylindrical chamber. In particular, for the noncompressible liquid ($\rho_c u = \text{const}$) when $D = \text{const}$ the behavior of \sum^2 for the nozzle and the cylinder should be identical. The compressibility effect leads to the fact that with the narrowing of the duct due to a decrease in the density, the velocity increases faster than for the noncompressible liquid. This leads, with the constant diffusion coefficient, to the retarding of mixing as compared with the mixing in a cylinder (Fig. 2, curve 2).

With the accuracy of the experiment, the mixing in a narrowing nozzle and cylindrical tube occurs equally. This is also confirmed by the absence of the stratification of the relative maximum excess stagnation temperatures (Fig. 3). (In constructing the graph, the results of the measurements were corrected in accordance with the values of the recovery factor found experimentally in the cross sections.)

Fig. 2. The mixing of a weakly heated jet in a narrowing nozzle: 1 - calculation for rapid deformation; 2 - calculation for gradual deformation; points - experiment.

Fig. 3. Change of maximum stagnation temperatures with mixing in the cylinder and nozzle.

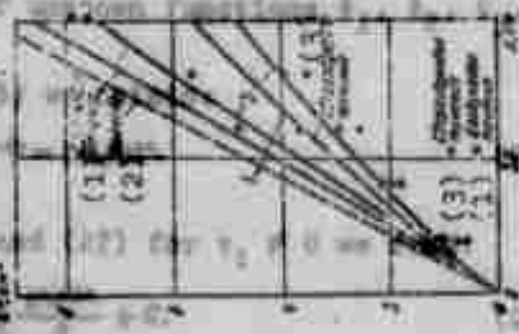
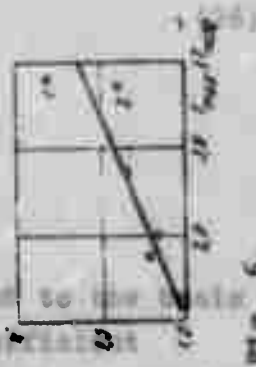
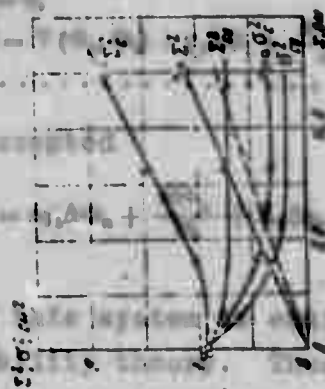
KEY: (1) Cylinder; (2) Nozzle.

Fig. 4. Mixing in the expanded part of a Laval nozzle: 1 - calculation for rapid deformation; 2 - calculation for gradual deformation.

KEY: (1) Subsonic flow; (2) Cylinder; (3) Supersonic flow.

Fig. 5. Mixing of a nonisothermal jet ($T_{\max}/T_{\min} = 2.7$) in a narrowing nozzle (broken line - the results of experiments in a cylinder).

Fig. 6. Effect of the nonuniformity of temperatures on mixing in a narrowing nozzle: 1 - nozzle; L = 200 mm; 2 - nozzle; L = 400 mm; points - experiment.



The experiments carried out in the expanded part of a Laval nozzle (cone 400 mm long; diameters of entrance and exit 85.3 mm and 90.0 mm; subsonic part a with a length of 200 mm, Vitashinskiy profile) with pressures in the chamber which ensure supersonic ($M = 1.0-1.4$) and subsonic ($M = 0.79-0.62$) conditions showed that in this case the retarding of mixing due to compressibility is more considerable, especially for the supersonic conditions (Fig. 4).

As follows from the graph, the experimental points lie nearer to the curves calculated under the assumption of gradual deformation ($D = \text{const}$). (To explain the actual mechanism for the deformation of turbulence with flow in the nozzles, it is necessary to perform the measurements of the fluctuating characteristics of velocity which behave substantially differently in the case of rapid and gradual deformation.)

We note that the mixing in a cylindrical duct corresponds to a value of the coefficient of turbulent diffusion several times larger than with established tube turbulence (the dotted lines in Figs. 2 and 4 correspond to $D/2r_{\text{H}} u_{\text{H}} = 0.006$ while for the established tube turbulence $D/2r_{\text{H}} u_{\text{H}} = 0.0009$ with $Re > 10^5$ [4]). This, apparently, is connected, apart from the large blocking of the flow (pylons, grids, etc.), with the presence of a boundary layer on the feeding tube which intensifies the mixing process of the jet.

During the analysis of the experiments, the source of the heated gas cannot be considered as a point source. In this case, it is possible to show that with a one-dimensional flow the distribution of concentration (excess stagnation temperature) over the cross section is described by the solution of the diffusion equation for a circular source which, with the insignificant effect of the walls on the concentration, has the following form:

$$C(r, x) = \frac{C_0}{\sqrt{4\pi D x}} \exp\left(-\frac{r^2}{4Dx}\right) \quad (4)$$

where $a(x) = a_0 r_0(x)/r_{\text{H}}$; $a(x)$ is the radius of the jet in the current cross section of the nozzle and a_0 - the initial radius of the jet.

For determining the value of dispersion from the profile described by the P-function, from the equation which follows

$$\Delta T_{\max} \Delta T_0 = 1 - \exp[-a(x)/2\sigma(x)],$$

also does not depend on time.

directly from (4), the ratio a/σ was found according to maximum temperature. The value σ was determined from the width of profile of the perturbation b with $\Delta T = \Delta T_{\max}/2$. Used for this was the dependence of b/σ on a/σ which was obtained from the data of [5], where the P-function is tabulated. For the isothermal flow, dispersion can also be found directly from (5) if we make use of the expression for $a(x)$.

For investigating the mixing of a nonisothermal jet, a unit was used which differs from heat depicted in Fig. 1 in the fact that the precombustion chamber was installed coaxially within the cylindrical chamber and the tube with length ≈ 0.3 m for the feeding of the jet was smooth (without heat insulation). This led to the fact that the slope of the straight line $\sum^2(x)$ with mixing in the cylinder corresponded almost half as much to the value of the coefficient of turbulent diffusion than in the preceding experiments.

In our case, from expressions (29) and (30) it is not difficult to find the temperature of the jet was about 2000°K, and nonisothermicity at the nozzle entry was controlled by a change in the distance between the beginning of the jet and the nozzle entry (in the experiments: $L = 170$ mm, 300 mm, 450 mm). Experiments were carried out both in Vitoshinskiy nozzles with a length of 200 mm and 400 mm, with the above indicated cross sections with a critical pressure gradient and in a cylindrical compartment. For an example, Fig. 5 gives the results of experiments in a nozzle with a length of 200 mm with $L = 300$ mm. The amount of dispersion of the experimental profiles of the excess stagnation temperature which were approximated by the P-function was found from the width of the profile and the maximum temperature, as indicated above. A change in the dispersion along the nozzle in the absence of mixing was found from formulas (1) and (2); in this, the profile at the nozzle entry was replaced by Gaussian curve with some effective parameters a_0 and σ_0 - such that the maximum excess temperature and the width of profile would

coincide respectively. From the parameters for Gaussian profiles calculated along the nozzle the dispersion σ_{co}^2 and \sum_{co}^2 was determined for the profiles described by the P-function and having Gaussian width and maximum temperature, and finite perturbations with the amplitude $A_1 < 1$.

In the case of flow ($\gamma_1 > 0$) the initial laminar flow will be perturbed with respect to the initial and finite perturbations with amplitude $A_1 < 1$. The initial flow will approach a low level of turbulence only to the phase plane.

During the derivation of (29) and (30), to ensure convergence of series (2) the condition $A_1 < 1$ was assumed. Thus, the vicinity of point $A_1 = 1$ still requires, strictly speaking, supplementary investigation. It is possible to express some plausible assumptions and to obtain results still another interpretation. As can be seen from Fig. 12, the new laminar flow which corresponds to Fig. 8(1; 0) of the phase plane will, with $\gamma_1 > 0$ and with the given number, poss. Fig. 7. Temperature profiles with mixing in a narrowing nozzle.

Fig. 7. Temperature profiles with mixing in a narrowing nozzle. Fig. 8. The dependence of the parameters of mixing with a low level of initial turbulence: to the perturbations with amplitude $A_1 > 1$.

1 - $\Delta T_0 = 1200^\circ$; $\Delta T_{max}(0) = 720^\circ$	Experiments in the nozzle; \sum_{co}^2 was determined from b and ΔT_{max} with respect to infinite and the finite perturbations. However, they all are based on the "analytic" expression (2).
2 - $\Delta T_0 = 1600^\circ$; $\Delta T_{max}(0) = 240^\circ$	
3 - $\Delta T_0 = 160^\circ$; $\Delta T_{max}(0) = 75^\circ$	
4 - $\Delta T_0 = 160^\circ$; $\Delta T_{max}(0) = 76^\circ$	Experiments in the cylinder; \sum_{co}^2 was determined from ΔT_{max} even reliably evaluating the first expansion coefficient a_1 .

As follows from Fig. 5, despite the presence of mixing in conclusion we will dwell on several results. In $[\sigma_c(x) > \sigma_{co}(x)]$, the absolute value of dispersion in the nozzle decreases. It is interesting to note that the experimental values of $\sum_{co}^2(x)$ practically lie on a straight line. Figure 6 gives the dependence of the relation of the tangents of the angles of slope of straight lines approximating experimental values of \sum_{co}^2 in the unstable region it led to the initial increase in \sum_{co}^2 with time. This deficiency was unessential until the attempt was

nozzle and cylinder on the relation of the maximum and minimum temperatures at the entry to the nozzle. From the graph it follows that with an increase in the nonuniformity of temperatures the angle of slope increases, which indicates additional turbulence with flow in the nozzle.

Experiments were also carried out in the study of the process of the intensification of mixing with a nonisothermal flow in a nozzle with a reduced level of initial turbulence. For this purpose, on the unit depicted on Fig. 1, the entire flow before the entry nozzle was passed through a honeycomb having ducts with a diameter of 2.5 mm and prepared from a long corrugated metal strip 0.5 mm thick and 40 mm wide. Figure 7 gives as an example the temperature profiles measured by differential thermocouples in a nozzle with a length of 400 mm with the initial overheating of the jet $\Delta T_0 = 160^\circ$ and $\Delta T_0 = 1200^\circ$ (on the nozzle edge $M = 0.87$; overheating was controlled by changing the flow rate of the combustion products from the precombustion chamber and air being mixed). From the drawing one can see that with great overheating the temperature falls more intensely. Figure 8 gives values of Σ with mixing in a cylinder and nozzle with different overheatings; the great effect of the nonisothermicity of the flow on the mixing is evident.

It is interesting to note that even in the case of small overheating in the nozzle the relatively small agitation of the flow takes place with damped turbulence which is imperceptible with large levels of turbulence (Fig. 2). This, apparently, is connected not only with the nonuniformity of velocities (which can attain 5-7%) due to nonisothermicity but also with the longitudinal velocity shift with the deviation of actual flow in the nozzle from a one-dimensional one.

The author thanks L. D. Kulikova who took a large part in the conduct of the experiments.

From what has been presented, it can be seen that with a small degree of turbulence of the incident flow the transitional zone

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ON THE MIXING OF COCURRENT JETS OF GASES OF DIFFERENT AND IDENTICAL DENSITY

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The presently-existing semiempirical theories of free turbulent flows expounded in [1-3] are, in essence, differential methods of processing experimental data. Furthermore, measurements of the turbulence structure do not confirm the models forming the basis of these theories. Therefore, they cannot be used in cases not first investigated experimentally. Some information about free turbulent flows can be obtained using a dimensional analysis, considerations of symmetry, laws of conservation of mass and momentum, and also the experimental data on the structure of turbulence. But the full picture of the mixing of two flows can be obtained only on the basis of a correctly posed experiment.

In view of the large number of parameters which affect mixing in a specific experiment and their simultaneous change, the isolation of dependence on a specific parameter, for example on the relationship of velocities, is not always possible. In many works on the jet flows changes in the initial conditions are not considered, which leads to different forms of the dependences being obtained.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \nu \nabla^2 \omega \quad (1)$$

Figure 1 gives the diagram of a working section. Central nozzles with a diameter of 40 mm and 30 mm and thickness of the edge of 0.4 mm were fastened to two pylons in tubes with a diameter of 300 mm and 120 mm respectively. For the deturbulence of the flows and change in conditions, at the entrance in the cocurrent flow and in the central nozzle fine-pored grids with the dimension of mesh from 0.07 mm to 0.3 mm and porosity from 0.35 to 0.5, respectively were installed. The measurements of the coefficient of turbulent diffusion D_T in the air flow behind such grids according to the measurements of the expansion of the thermal trace behind a heated wire with a diameter of 0.03 mm with the air speed of 15 m/s and normal temperature showed that it is close in value to the molecular coefficient of diffusion.

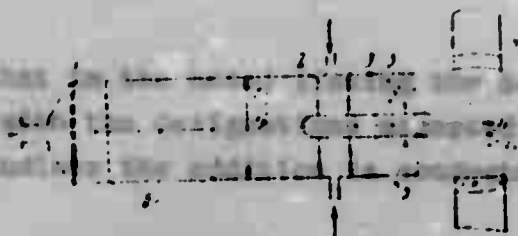


Fig. 1. Diagram of the working part of the unit. 1 - grid and felt, 2 - pylon, 3 - central nozzle, 4 - shadow instrument IAB-451, 5 - grid on the edge, 6 - fine-pored grid.

The setting of the general grid (or package of two-three grids) at the nozzle edge of the working section (see Fig. 1) effectively equalized the velocity profiles in the boundary layers on the edge. Setting the grids at different distances from the edge, it is possible to obtain the different dimensions of the boundary layers on the edge which determine the initial conditions of the experiment.

The geometric characteristics of the flow - the width of the zone of mixing and the length of the "nucleus" - were determined

from the measurements of the temperature profiles and dynamic pressure. Taken as the boundary of the zone of mixing¹ were points of the profiles in which the relative excess of temperature ΔT or dynamic pressure $\Delta(pu^2/2)$, or velocity Δu equal 0.97 and 0.03. For the measurement of the temperature profile one of the flows was heated. The temperature drop between the flows was not more than 50°C. The measurements of the temperature fields in the flow were carried out with the use of chromel-copel thermocouples assembled in a comb of 30 pieces with the distance between them of 3 mm. The diameter of the joint of the thermocouple was 0.1 mm. The readings of the thermocouples were recorded on our EPP-09 recording potentiometer. The time for reading of one profile was 15 s. The profiles of the dynamic pressure were measured by a comb of 25 total pressure tubes whose readings were taken on an inclined multitube pressure gauge. Furthermore, on the nozzle edge the gas currents of different density were photographed by the shadow method or with the use of a Toepler tube. Velocities were changed in a range of 10-50 m/s. Air, helium and freon-12 were used as the working media.

The boundary layer effect can be illustrated by Fig. 2, where the dependence of the ordinate of the internal boundary of the zone of mixing y_1 (according to the relative velocity $\Delta u = 0.03$) on the relation of velocities m of the cocurrent flows of air at distances from the nozzle edge $x/d = 7.5$ is given (this value is different at different distances from the edge and increases with distance from it). In the first case the grid on the edge was absent and, on the external wall of the central nozzle, the boundary layer at a length of 250 mm grew. In the second case, on the edge a grid was installed with the dimensions of mesh 0.07 mm and a porosity of 0.35. As can be seen from the graph, in the absence of a grid at the edge, mixing stops depending on the relationship of velocities, beginning with $m = 0.5$.

(7) It follows that the value of m at which the boundary of the zone of mixing is determined is specially stipulated.

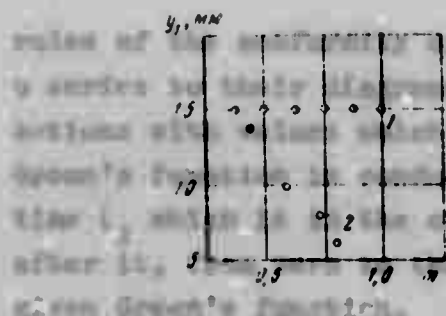


Fig. 2. Dependence of y_1 on the relation of the velocities of two jets with constant velocity of the central jet equal to 15 m/s; $x/d = 7.5$. 1 - without grid at the edge; 2 - with grid.

For homogeneous gases, the dependence of the width of the zone of mixing on the relation of velocities with a weak effect of initial conditions was investigated with grids installed on the nozzle edge. The boundary layer thickness in this case comprised less than 1 mm from each side of the edge. The corresponding dependence of the width of the zone of mixing determined from the relative excess velocity on the relationship of the velocities is given in Fig. 3. As is evident, the experimental results are well described by the linear dependence up to $m = 0.95$. In obtaining this dependence, it was considered that on a considerable part of the initial section of the jet the flow bears either a clearly expressed periodic character, or a transitional character to a turbulent one - this section was eliminated from examination. Its length was determined from the instantaneous photographs of the flow.

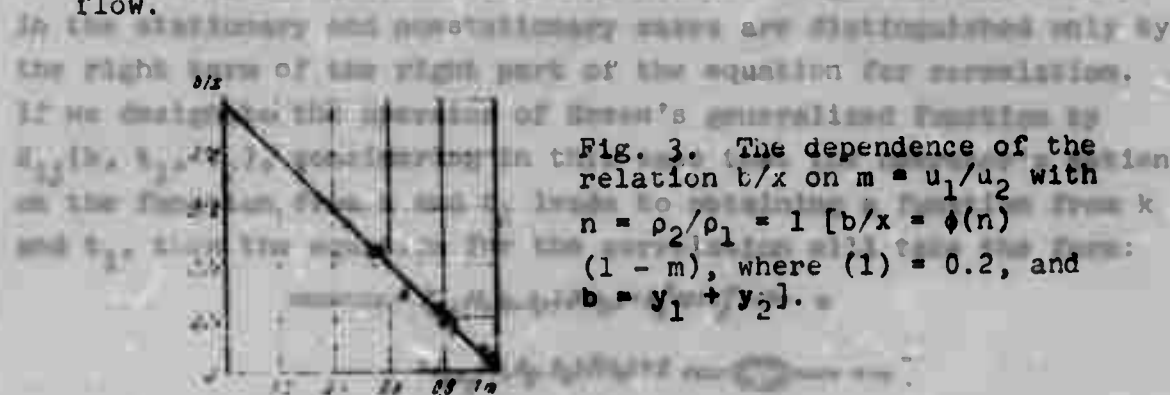


Fig. 3. The dependence of the relation b/x on $m = u_1/u_2$ with $n = \rho_2/\rho_1 = 1$ [$b/x = \phi(n)(1 - m)$, where $(1) = 0.2$, and $b = y_1 + y_2$].

The outlines of this diagram elements are presented in the last equation. In connection with the above indicated strong influence of the initial conditions (to which the relative misalignment of the jets is related), it is difficult to agree with Spalding in the fact that in the derivation of the equation for the generalized function

It is possible to introduce a second apical function without taking into account in this case the dependence of the apical functions on the wave number (the two flows should also pertain) and absence in different works of the necessary information about them, it is difficult to conduct a comparison of different experimental data. In some works a number of dependences of the width of the zone of mixing on relations of velocities m and densities n obtained under different assumptions relative to the model of the flow in the layer of mixing is proposed. Thus, in works [1] and [4] it is assumed that the relation of the width of the zone of mixing to the distance from the edge is proportional to the relation of the two velocities: the transverse displacement is proportional to the modulus of the difference in the velocities of the two flows $\Delta u = |u_1 - u_2|$, and longitudinal - to some average velocity for the zone of mixing determined according to the formula $u = \frac{\int u dy}{\int dy}$. In this work it is selected:

$$u = \frac{\int u dy}{\int dy}$$

Then we have: $b/x \sim \Delta u/u^*$. With the mixing of homogeneous flows this leads to the relation $b/x \sim (1 - m)/(1 + m)$ which is not confirmed in the experiments of the authors of this work. Also not confirmed is the dependence of the width of the zone of mixing on the density ratio for the submerged jets which follows from this relationship. From it it follows that with the mixing of a jet in the atmosphere of a lighter gas, the length of the "nucleus" cannot increase as compared with the mixing of homogeneous flows more than two-fold, and the width of the zone of mixing of the jet in an atmosphere of heavier gas exceeds considerably that obtained in the experiments. In the experiments of the authors the length of the "nucleus" of the submerged jet of freon in an atmosphere of air increases approximately 3.5 times.

From the above indicated assumptions it follows that the dependence of mixing on n is different for different m , i.e., with equal Δu but with different absolute velocities. However, it is more reasonable to assume that the features of the interaction of

the two flows will be determined only by their relative motion if, of course, mixing is not influenced by the initial nonuniformities of velocity and initial turbulence. (This assumption, taking into account the experimental results, makes the foregoing dependences unacceptable.

If one assumes that the dependence of mixing on the density ratio with an equal difference in velocities does not depend on the absolute values of velocity, then for the width of the zone of mixing we obtain: [2] will be obtained.

$$b = (x - x_0)(1 - m)\eta(n).$$

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where the x_0 is the effective origin of the coordinates determined by the conditions of transition to the turbulent flow and $f(m)$ and $\phi(n)$ are some functions which depend respectively only on m and n .

The linear dependence of the width of a flat zone of displacement on x is obtained from dimensional analysis, but dimensional analysis provides no information relative to the type of function $f(m)$ and $\phi(n)$. However, if one assumed that the mixing is entirely determined by the difference in the velocities, and the lesser of them accomplishes a shift of the picture of flow in the flow direction, then we obtain:

$$b = (x - x_0)/m\eta(n).$$

In the derivation of this dependence it was assumed that with an identical difference in velocities the dimensions of the zones of mixing with flows with different absolute velocities are equal with the same values of coordinate $x - x_0$, in which the longitudinal mixings of unperturbed particles of one flow are equal relative to the unperturbed particles of the other. The linear dependence of the width of the zone of mixing on the relation of velocities obtained under such assumptions is confirmed well experimentally with $n = 1$ (see Fig. 3). From these assumptions

it follows that the dependence of the width of the zone of mixing on the density ratio is identical with any relationship of velocities. Therefore, the experimental determination of the function $\phi(n)$ can be conducted with any relation of velocities, including in submerged jets ($m = 0$), which was also done by the authors.

The study was carried out in the range of the density ratio $1/n$ from $1/30$ (jet of freon in an atmosphere of helium) to 30 (jet of helium in an atmosphere of freon). The central jet was heated, and the width of the zone of shift was determined from the temperature profile. For a decrease in the heat exchange and temperature boundary layers on the dividing edge the flows were heat-insulated with the aid of a thermal insulation case slipped over the central nozzle. Measurements in each cross section were carried out on the edge of the tube. In individual experiments the value of m was somewhat more than zero, but it never exceeded 0.05 . A turbulent boundary layer was open in the sense that the scale of turbulence was given in the form of an empirical function of coordinates. As can be seen from Fig. 4, the intensity of the mixing of a heavy jet in the atmosphere of a light gas decreases sharply in comparison with the intensity of the mixing of homogeneous flows ($n = 1$), and with a decrease in the jet density as compared with the cocurrent flow the intensity of mixing increases very weakly. This can be explained by the fact that the intensity of mixing is determined by processes on the interface between turbulent and nonturbulent liquids, on which the density ratio is changed sharply with the blowing-in of a heavy jet into the atmosphere of a lighter gas and weakly in the opposite case. Measurements at adjacent points cannot be arbitrary due to viscosity. Therefore, the two-point correlation Figure 5 presents the profiles of the relative dynamic pressure and relative temperature for a jet of freon in an atmosphere of air. Clearly, it can be seen that the profile of dynamic pressure is narrower than the temperature profile in the

(1.1)

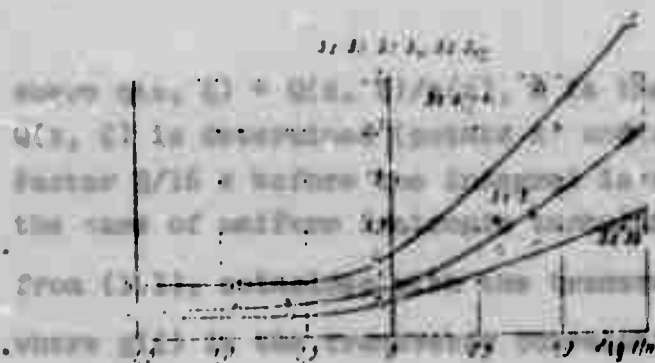
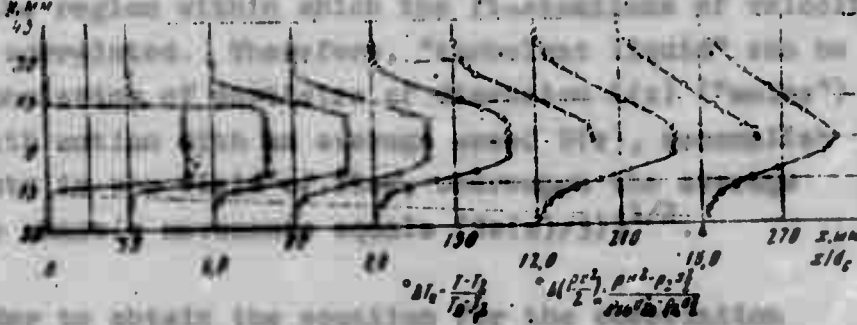


Fig. 4. Dependence of the width of the zone of mixing on the density ratio $\rho_{\text{arm}}/\rho_{\text{jet}}$ with $m = 0$.

The laminar integral scale is a characteristic of the linear dimensions of the region within which the fluctuations of velocity are noticeably reduced. The velocity profiles at different distances from the nozzle edge are presented in Fig. 5, which, along with the velocity profiles, shows the profiles of the relative temperature and the relative dynamic pressure of the jet.



2. In order to obtain the equation of the function $Q(x, y)$, we use the velocity profile $u(y/2)$, and let us assume that the velocity profile is given by the equation, for the case of a submerged jet of freon in the air (average velocity $u_{10} = 10.4$ m/s, $T_{10} = 310^\circ\text{K}$).

exterior portion of the zone of mixing, whereas in the inner part of their boundaries they coincide, i.e., the velocity profile is narrower than the temperature profile whose boundaries coincide with the boundaries of the profile of concentrations. Consequently, in the case of different densities the velocity profile is narrower than the profile of concentration, too. This leads to the nonmonotonic change of the profile of dynamic pressure in the zone of the mixing of two flows of different density when the difference in dynamic pressures is small and the dynamic pressure in a light gas is greater than in a heavy gas. Figure 6 gives the profiles of dynamic pressure and temperature in the zone of

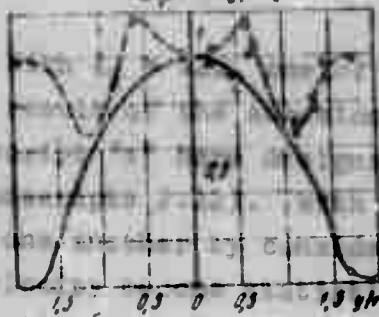


Fig. 6. Profiles of dynamic pressure and temperature with equal dynamic pressures of the central jet of helium and cocurrent flow of air ($x = 102$ mm, $u_{\text{air}} = 14.2$ m/s).

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us integrate for the region Ω . In the case of the flow around the surface region, Ω is the zone limited by planes $\xi_2 = \pm 2x_2$.

Although region Ω depends on x_2 , in the integration of terms of the type $\partial^2 Q_1 / \partial x_2^2$ it is possible to carry out the sign of the derivative as the integral sign, since the surface integrals being obtained in this case are equal to zero due to boundary conditions (2.2).

Let us assume that

$$\psi(x, y) = \psi_1(x, 0) \psi_2(y, z)$$

T. 10000

$$\frac{1}{\Gamma} \frac{\partial}{\partial t} \left(\frac{\partial \rho}{\partial t} \right) \frac{\partial^2 \rho}{\partial t^2} = \frac{1}{\Gamma} \frac{\partial}{\partial t} \left(\frac{\partial \rho}{\partial t} \right) \frac{\partial^2 \rho}{\partial t^2} \quad (2.3)$$

at 17

$$x_i = \frac{1}{n} \sum_{j=1}^n x_{ij} = \frac{1}{n} \sum_{j=1}^n (a_{ij} + d_{ij} + \dots + c_{ij})$$

Let us designate $T_{\Sigma} = 2\pi(\partial Q/\partial E_{\Sigma}) + (u_{\Sigma}^2 + u_{\Sigma}^{\prime 2})(\alpha' - \alpha'')/2$. Then dissipative term \dot{Q}_{Σ} of equation (5.1) can be presented in the form

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Let us present for an example the formulas for the calculation of the layer of mixing between two plane-parallel nonisothermal flows. Let the density, velocity and temperature of the first and second flows equal respectively, ρ_{01}, v_1, T_1 and ρ_{02}, v_2, T_2 . The average partial densities are equal to ρ_1 and ρ_2 . The effect of the adhesion of the liquid to the fairing.

With the usual assumptions, the mixing is described by the equation of turbulent diffusion of the form (1) During the time Δt the turbulent mole from point x will move with the middle flow to point $x + U\Delta t$ where it will have a cross-sectional region $\lambda(x + U\Delta t)$. If $\frac{\partial \rho_1}{\partial x} + \frac{\partial \rho_2}{\partial y} + \frac{d}{dt} \frac{\partial \rho_1}{\partial y}$, rate of growth of the region of a mole

(where \bar{u}_{cp} - the average longitudinal velocity in the layer of mixing), whose solution with initial conditions: we will obtain the left part of equation (2.5). Thus, the left part of (2.5) expresses the rate of growth of the average region of the moles.

and boundary conditions: With the chaotic movements the moles capture the adjacent particles of liquid by the force $y = -kx$; ρ_1 increases and increases in dimensions. This effect is expressed by the penultimate term of equation (2.5) has the form $\frac{d}{dt} \frac{\partial \rho_1}{\partial y}$, close to the fairing $L \sim x_2$ the region of the variable ξ in which $Q(x, \xi)$ is determined is limited by planes $\xi_2 = \pm 2d_2$. Therefore, the dimensions of the moles close to the fairing cannot increase boundlessly and they should be limited by a value proportional to x . The fairing reduces the rate of increase of the dimensions of the moles. This effect is expressed by the last term of equation (2.5).

The integral laws of conservation of the flows of substance and momentum written on the assumption that the transverse velocity of the averaged flow is continuously fed to the moles. This when $y = -\infty$ is equal to zero are the following: process also affects the rate of growth of the dimensions of the moles and is expressed by the sixth term of equation (2.5).

$$\frac{d}{dt} \int_{-\infty}^{\infty} u \rho_1 dy = 0. \quad (4)$$

$$\frac{d}{dt} \int_{-\infty}^{\infty} (u^2 + v^2) dy = 0 \quad (5)$$

Terms are numbered taking into account the left part of the equation.

They make it possible to establish the following connections between the diffusion parameters: the dimensions which correspond to any point of the field of flow fall into adjacent points. Such movement $\frac{a_r}{\sqrt{c^2 + z^2}} = (1-m)\Psi(x) = 0$, transfer of scale by means of turbulent diffusion is expressed by the fifth term of equation (2.5).

$$\Psi(x) = \frac{ny + (1-m)(1-n)\Psi(y) - ma\sqrt{\frac{Pr}{\pi(1+\frac{z^2}{x^2})}}}{\sqrt{c^2 + z^2}} \quad (7)$$

Finally, because of viscosity the $\frac{1+Pr}{\pi(1+\frac{z^2}{x^2})}$ stochastic movements, can lose a part of the $\sqrt{c^2 + z^2}$ belonging to them, transferring it to the scales located in the adjacent points of the field where $m = v_1/v_2$, $n = \rho_{01}/\rho_{02}$. Furthermore, the designations are the same. "Intolerance" arises which is expressed by the third and fourth term of equation (2.5).

$$\Psi(x) = \frac{x}{2} [1 + \Phi(x)] + \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad x = \frac{a_r - \bar{a}_r}{\sqrt{c^2 + z^2}}, \quad \bar{a}_r = \frac{e^{h^2 z_r}}{1 + \Phi(\frac{1}{\sqrt{\pi}})}$$

In the case of isotopic turbulence with small r

$$y = x \sqrt{\frac{1+Pr}{1+\frac{z^2}{x^2}}} + \sqrt{\frac{Pr}{\pi(1+\frac{z^2}{x^2})}} = \dots$$

If we substitute this formula into the expression for c , then it turns out that Formulas (6) and (7), in the particular case $n = 1$ give the according simple relationship of Notta [2]. For σ_{12} Notta [2] gives the following estimate $\sigma_{12} = 0.12$. We will assume that σ_y and σ_{12} are absolute constants. $\sigma_y = \frac{1+m}{\sqrt{\pi}} \sigma_y$ (7')

For the determination of the temperature profile the following In order to estimate B, let us examine equation (2.5) with "two-stage" model is used. First the quasi-laminar temperature small x . Since close to $x_2 = 0$ according to equation (2.5) profile with uniform mixing with dispersion σ^2 is found: $c/e \rightarrow 0$, if we reject the small terms equation (2.5) is simplified:

$$v_2 D \frac{T_2 - T_1}{T_2 - T_1 \rho_{1c} + \rho_{2c}} = \frac{P_{2c}}{n \rho_{1c} + P_{2c}}$$

From this expression and from the experimental distribution of the scale close to $x_2 = 0$ ($L = x_2$ see [1]), and also from formula (2.4a) it is possible to obtain the following approximate expression:

¹Its introduction is necessary during the study of the mixing of jets of substantially different densities and temperatures, and also to account for the nonuniformity of mixing during the calculations of combustion and radiation in jets. where $\rho_{1c} = 1$. The specific form of this function can be selected in such a way that the obtained calculated distribution of the scale close to $x_2 = 0$ coincides with the experimental.

3. We combine the system of equations obtained in [1] with equation (2.5). Then, using (2.6) and the numerical values of constants which enter these equations, we will obtain: the normal law with dispersion $\sigma_T^2 = \sigma^2 - \sigma_G^2$:

$$\begin{aligned} \frac{\partial T}{\partial x_1} + \frac{\partial T}{\partial x_2} &= -\frac{1}{\rho} \frac{\partial p}{\partial x_1} + \frac{\partial}{\partial x_1} \left(\nu \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\nu \frac{\partial T}{\partial x_2} \right) \\ \frac{\partial T}{\partial x_1} + \frac{\partial T}{\partial x_2} &= \frac{\partial}{\partial x_1} \left(\nu \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\nu \frac{\partial T}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left(\nu \frac{\partial T}{\partial x_3} \right) \\ \frac{\partial T}{\partial x_1} + \frac{\partial T}{\partial x_2} &= \frac{\partial}{\partial x_1} \left(\nu \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\nu \frac{\partial T}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left(\nu \frac{\partial T}{\partial x_3} \right) \end{aligned} \quad (8)$$

where

$$\frac{\partial T}{\partial x_1} + \frac{\partial T}{\partial x_2} = \frac{\partial}{\partial x_1} \left(\nu \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\nu \frac{\partial T}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left(\nu \frac{\partial T}{\partial x_3} \right)$$

where

Analogously it is possible to obtain the expression for the mean-square fluctuation of temperature:

$$\sigma_T^2 = \frac{\partial}{\partial x_1} \left(\nu \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\nu \frac{\partial T}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left(\nu \frac{\partial T}{\partial x_3} \right)$$

In general, the boundary conditions for system (3.1) will be the following. The comparison of the results of the calculation with the experimental data for a series of problems (circular jet, plane jet at the wall, distributed blowing-in from the walls, pressure behind an offset with the turning of a supersonic flow) is presented on Figs. 1-5. A diffusion model was used also for the calculation of diffusion with the flow of a jet in a nozzle, the Coanda effect, diffusion flames and other technical problems with the same satisfactory agreement with the experiment.

$$U = \frac{\partial}{\partial x_1} \left(\nu \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\nu \frac{\partial T}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left(\nu \frac{\partial T}{\partial x_3} \right) \quad (3.3)$$

Fig. 1. Change of temperature in a cocurrent circular jet; I. B. Palatnik's experiments (continuous lines - calculation; $\Delta T_0 = 25^\circ K$).

Fig. 1. Change of temperature in a cocurrent circular jet; I. B. Palatnik's experiments (continuous lines - calculation; $\Delta T_0 = 25^\circ K$).

System of equations (3.1) with boundary conditions (3.2) was integrated by the method of finite differences for the case of a flat plate ($\nu = \text{const}$, $\nu = 1.5 \cdot 10^{-4}$). The integration began in the laminar part of the flow from point $x = 1.5 \cdot 10^{-4}$. The initial distributions of average velocity, temperature and scale of turbulence were given in the following manner:

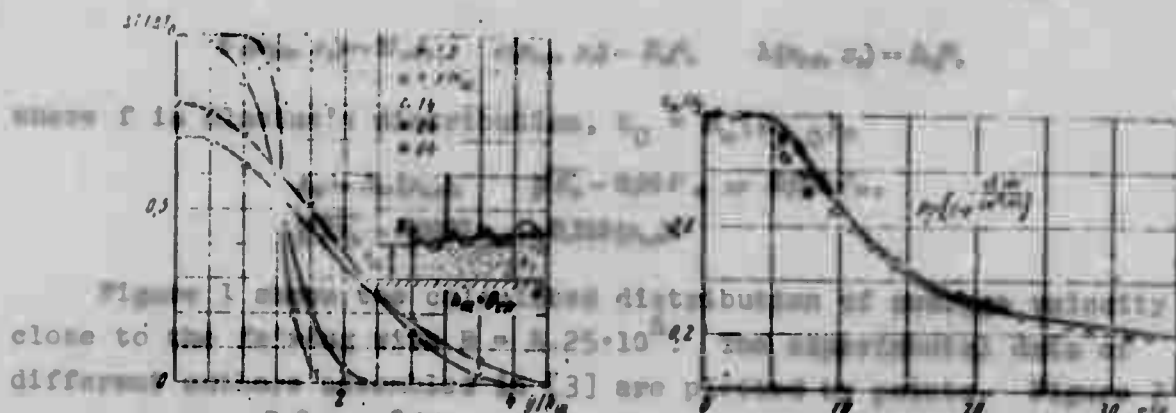


Fig. 2. Change of temperature in a cocurrent jet at a wall; V. Ya. Borodachev's experiments (continuous lines - calculation; $h_w = 5 \text{ mm}$, $\Delta T_0 = 690^\circ \text{K}$). worse with the experimental than close to the fairing.

Fig. 3. Axial velocity in a circular submerged jet. The dotted line - experimental data of Korsin for an air jet; the small circles are Forstoll's experimental data for a water jet; the continuous line - calculation with $\sigma/a = 0.095(x/d - 2)$, $Pr = 0.5$).



Fig. 4. Profiles of concentration for various blow-ins; experiments of I. V. Bessalov and A. M. Gubertov (continuous lines - calculation). KEY: (1) Axis of tube. c/r is value $x_2/\delta \leq 0.6$ $c/r = 0.2$ is obtained at $x_2/\delta = 0.6$ increases, reaching with $x_2/\delta = 6$ values on the order of 1.0 (the data were obtained by processing Klebanov's [4]).

Everywhere the turbulent number Pr was taken as $Pr = 0.5$. The degree of uniformity of mixing N , determined for the first time for gases by N. A. Zamyatina [8, 9] with the aid of the

optical-diffusion method, was taken as equal to $N = 0.4$ (gas - gas) and $N = 0$ (gas - liquid). The only determining parameter was the stream dispersion of the jet, σ in the following form:

$$\sigma^2 = -(\sigma_0^2) \exp(-\frac{x}{x_0}) \quad (3.4)$$

If we take experimental distributions (1) to $x_2 = \delta$ we will obtain a flat offset (dotted line - Korst's calculation: $\sigma^2/h = 0$; continuous - S. Kh. Oganessian's calculation according to the diffusion model of mixing: $\sigma^2/h = 0$). KEY: (1) Timoshin; (2) Kuptsov; (3) Tagirov. considerable deviation of the distribution from experimental data to $x/x_0 = 1$ will be obtained, whereupon we will obtain experimental values $(U_1 - U_2)/u$. At the same time it is evident that with $\sigma = 0$, $\sigma_0 = \text{const}$, we will obtain considerable deviations in the velocity distribution near the pairing.

The dispersion of a jet on the basic section depends only on relationship of velocities (the parameter of cocurrence m) and for all the above-enumerated problems proves to be identical with the identical value of parameter m with an accuracy up to the length of the initial section. All other factors - difference in densities, initial turbulence, twisting of the jet, boundary layer thickness, acoustic effects, etc., - affect basically only the length of the initial section of the jet. For example, for the subsonic velocities dispersion on the basic section satisfies the following calculated relationship:

under the same conditions (points are Klebanov data from [4]). The deviation of the calculated distribution $\sigma^2/\sigma_0^2 = k^2(x - x_0)^2$ from experimental can be (9)

explained by two reasons: first, the greater values of $\partial U_1 / \partial x_2$ at $x_2 = 2x_0$. Here σ_0 is dispersion with $m = 1$; $\sigma_0^2 = \epsilon_1 x^2$ with $x < 2x_0$; and therefore by the greater generation of the energy of turbulence in comparison with $x_1(x - x_0)$ with $x \geq 2x_0$, where $x_1 = D_1 / 2v_1 \epsilon_1$; $D_1 / v_1 d_1 = 0.0009$, and ϵ_1 , D_1 , v_1 - the level of turbulence, the diffusion coefficient, and velocity of the external flow respectively; $x_0 = \sigma_0^2 / \epsilon_1$ for $m = 1$ and the initial tube profile of velocity; d_1 - the tube diameter; d_0 - the diameter of the jet, and the values of the boundary layer. The most reliable data on the value of the dissipation of the energy of turbulence in the boundary layer were obtained by

of k can be approximated by the dependences: $k = k_T(1 - m)$, $k_T = 0.09$ for $m < 1$ and $k = k_T(1 - m)$, $k_T = 0.06$ for $1 < m \leq 2$.

However, in the diffusion model, as in all semiempirical models, the characteristics of mixing are used which are borrowed from the experiment; this model does not make it possible to disclose the physical nature of the diffusion of a jet and, consequently, in principle it does not make it possible to theoretically determine the dispersion of a jet. Furthermore, this model does not provide a satisfactory description of the velocity field in the examination of semibounded jets and the boundary layer and is completely unsuitable, just as any other semiempirical models, for the "closing" of the equation of the conservation of energy (with the calculation of losses of total pressure).

Therefore, more promising (for the development of semiempirical theories of the "old type") it seems to us, is another, so-called vortex model. We note that recently a deeper penetration into the vortex nature of turbulent flows with a transverse shift is characteristic of many investigators [10, 11].

Figure 6a presents a diagram of a proposed vortex model for a free turbulent layer. The wave amplitude of the perturbation of a tangential velocity discontinuity increases up to the value of the order of the radius of the vortex [$\zeta(t) \approx l_0$], whereupon the wave is rolled up into the vortex. The nature of the change in ζ in the course of time is presented in Fig. 6b; the simplest approximation of the dependence $\zeta(t)$ is given. A more precise approximation, taking into account nonlinearity according to Landau [12], will not change the essence of subsequent reasonings. On curve $\zeta = \zeta(t)$ (Fig. 6b) it is possible to note two characteristic scales: the lifetime of the discontinuity τ_p and the time of the vortex formation $\tau_{v.f.}$, which satisfy the following relationships:

In conclusion the author expresses his thanks to G. I. Tolstoy and V. B. Anisimov for assistance in the calculations. (10)

where λ_T is the increment of an increase in the disturbance wave in the course of time in the coordinate system connected with the wave. The transition to the corresponding scales of length in a motionless coordinate system is evident: $x_p = v_{\phi} \tau_p$ and $x_{0.8} = v_{\phi} \tau_{0.8}$ (v_{ϕ} - the phase velocity of a wave of a "drift"-type disturbance, τ_0 - the amplitude of the initial disturbance of the discontinuity surface). The value of τ_0 , just as the scale τ_p , is a random function on the strength of which the scale τ_p is also a random function. We will obtain its average value, substituting the expression τ_0 given in [12] through the fluctuations of velocity on the initial section and averaging:

$$p', p'' - \text{fluctuation of pressure at points } x', x''; \quad (11)$$

$$\tau_p = \frac{x_p}{v_{\phi}} \ln \left(2 \sqrt{1 + \frac{\Delta v^2}{v_{\phi}^2 + v_{\phi}^2}} \right).$$

P - mean pressure at point x ;

where λ_x is the increment of an increase in the disturbance wave of the "drift" type. Experiments [16] show that the maximum level of "noises" under normal conditions does not exceed $\epsilon_1 = \epsilon_2 = \epsilon_{\max} = 0.25$, so that $x_p/x_{0.8} \approx 3$.

δ - boundary layer thickness;

X - coordinate of the firing;

ϵ - dimensionless coefficient of turbulent energy;

ν - dimensionless coefficient of turbulent viscosity;

ν_{eff} - dimensionless coefficient of turbulent diffusion of the firing or turbulence;

ν_{eff} - dimensionless coefficient of turbulent diffusion of the firing;

ν_{eff} - dimensionless coefficient of turbulent diffusion of the firing;

Fig. 6. Vortex model (free layer of mixing). 1 - velocity profile in a large vortex; 2 - velocity profile in a trace of breakdown; 3 - increase in amplitude of perturbation of the discontinuity surface; 4 - model approximation.

The vortex as a single liquid volume (mole) is separated from the remaining flow by an unstable discontinuity surface; for a vortex there are two characteristic scales: the lifetime of the vortex τ and the time of its breakdown $\tau_{p.s.}$ and, correspondingly, scales $x_s = v_s \tau$ and $x_{p.s.} = v_s \tau_{p.s.}$ (v_s - the speed of movement of the center of mass of the vortex). By virtue of conditions (10) the vortex formation occurs almost instantly, which is equivalent to the inelastic collision of two liquid volumes. The laws of conservation of moment, momentum and the position of the center of mass for a vortex up to and after "collision" provide for a free boundary layer (center of the disturbance wave lies on the x-axis, see Fig. 6a):

$$\omega = \frac{\rho_1 \rho_2 l_1 \Delta u}{\rho_{cp} l_1^2} \quad (a), \quad v_s = \frac{\rho_1 m_1 + \rho_2 m_2}{\rho_1 + \rho_2} \quad (b), \quad y_c = \left(\frac{\rho_1^2 - \rho_2^2}{\rho_1 + \rho_2} \right) \frac{l_1}{2} \quad (c), \quad (12)$$

where $\rho_{cp} = \rho_1 + \rho_2/2$, l_1 is the radius of inertia of the vortex and ω is the angular velocity of the vortex.

For the near-wall layer (center of vortex lies at distance l_s from wall):

$$\omega = \frac{l_2 v_1}{l_1 l_2} \quad (a), \quad v_s = \frac{\rho_1 m_1}{\rho_1 + \left(\frac{l_2}{l_1} - l_2 \right) \rho_2} \quad (b), \quad y_c = l_s \quad (c), \quad (13)$$

where ρ_1 and v_1 are respectively the density and the flow rate far from the wall, ρ_2 - density near the wall, l_2 - the momentum thickness in the boundary layer up to the formation of a large vortex, and ω - the angular velocity of the vortex.

From the law of conservation of energy it is also possible to determine total pressure losses in a large vortex and trace of breakdown. From relationships (12) and (13) it can be seen that in a free layer a large vortex moves practically without slipping (the profile of longitudinal velocity takes the form shown on Fig. 6a), and in the boundary layer (since $l_1 \sim l_s \gg l_2$) the

vortex moves with slippage, forming a laminar sublayer from below and from above. The profile of the volumetric concentration in this case coincides with the profile of longitudinal velocity (Prandtl turbulence number in the vortex equals unit: $Pr_p \approx 1$; Fig. 7a).



Fig. 7. Diagram of mixing in a large vortex and trace of breakdown. 1a - the distribution of volumetric concentrations; 1b - the distribution of longitudinal velocities; 2 - mathematical expectation of intermediate compositions; 3 - field of instantaneous concentrations. KEY: (1) Large vortex; (2) Trace of breakdown.

The work [11, 12] on determining statistical characteristics of a fluctuating pressure on the surface in the turbulent boundary layer of a noncompressible liquid were based on the Polson [radial] equation for the cascade breakdown of a large vortex into a series of fine vortices with uniform and isotropic distribution (zero moment relative to the center of mass) provides the so-called trace of breakdown (turbulent spot) of the same scale l_p . The problem of the determination of the properties of the turbulent spot in many ways is identical to the problem of the breakdown of turbulence behind a grid. Therefore, the mean-square rate of fluctuations in the turbulent spot $\sqrt{v_t^2}$ in time t after the breakdown of a large vortex and the coefficient of microturbulent viscosity (diffusion) in the turbulent spot $\nu_t \approx D_t$ can be taken from the experiments [11, 13]:

However it proves to be that a simple connection is established between the fluctuations $\sqrt{v_t^2}$ on the surface and velocity fluctuations in the boundary layer. In order to be consistent with the average velocity of the motion of the trace of breakdown (\bar{v}_p) in the turbulent boundary layer of the incompressible liquid on an infinite plane $x_2 = 0$ (coordinates x_1 and x_3 lie in the plane of the surface). Flow in this layer

The profile of the average concentration (temperature) in the trace of breakdown is not reconstructed but continues to expand with time under the action of microturbulent gradient diffusion with coefficient D_{MT} . The profile of longitudinal velocity can be reconstructed so that according to the laws of conservation of the quantity and momentum of momentum the upper part of the volume of a large vortex which has broken down will move at the velocity of the upper flow ($v_{p1} \approx V_2$), and the lower - with the velocity of lower flow ($v_{p2} \approx V_1$), i.e., the field of longitudinal velocities after breakdown in principle can return to the initial state (curve 1b on Fig. 7b). The average profiles of concentration (temperature) and velocity at an arbitrary point of a turbulent layer are determined by the type of the function $p(l)$ - the density of distribution of the probability of scales l , and also by the probability of the appearance of a large vortex in an arbitrary cross section x of layer P_p , or by the probability of the appearance of a trace of breakdown P_b . Function $P_b(x, Re_\Delta)$ [or $P_p(x, Re_\Delta)$] with small x is sufficiently complex. However with an increase in x , where the number $Re_\Delta = \Delta u l / \nu$ will become sufficiently large, and the probability of the appearance of laminar spots is sufficiently small, these functions take the form and, furthermore, it is proved that with a sufficiently smooth surface $S(x, y, z)$ velocity \vec{v} is continuous in a closed region to the second order derivatives in x, y, z and pressure p possesses the continuous derivatives of the first order right up to the boundary. Estimates according to a number of indirect experimental observations and also according to some considerations which follow from linear theory, permit assuming that P_p and P_b are values of (2) and (3) order, i.e., $P_p \approx P_b \approx 0.5$. With $x \rightarrow +0$ from equations (2) we obtain three relationships which connect the pressure gradients with the second derivative of the velocity on the surface:

$$P_p = P_b \approx 0.5 \quad (15a)$$

The function $p(l)$ in type is close to a Maxwellian distribution; it is either proportional to the function of the increment of an increase in the perturbations of the "drift" type of wavelength L

(i.e., $p(l_g) = \lambda_x(a \cdot L)$ - a coefficient of the order of unity; preliminary estimates according to [16] give $a \approx 0.3-0.5$), or it is a function of this increment (depending on the type of spectrum of initial perturbations). The features of curve $p(l)$ are such that the greatest probability will be with scale l_m with the increment close to λ_{\max} which decreases with an increase in the microturbulent viscosity for the envelope $\lambda_x = (\lambda_m a' u_{\text{eff}}(n-1)) / (l_m)$ - the curve of the increment for an infinitely fine surface of a tangential discontinuity (see Fig. 6c). By virtue of this the mean value of the scale $\bar{l} \approx l_m$ for a free layer satisfies the following relationship:

$$\frac{\Delta \bar{l}_n(x)}{\nu_T} \approx \frac{1}{\text{Re}_\lambda} \frac{\Delta u_{\text{eff}}}{\nu_T} \approx \frac{1}{\text{Re}_\lambda} \frac{\Delta u_{\text{eff}}}{\nu_T} \quad (7) \quad (16)$$

The linearity of the right part of relationship (7) gives it a sense whereupon $\Delta \bar{l}_n(x) / \nu_T \approx 250-290$ - according to the data of [13]; $\text{Re}_\lambda = \Delta u_{\text{eff}} / \nu_T \approx 380$ - according to data for processing solutions of [14] for the case $n = 1$; $m = 0$ and the laminar thickness of the only discontinuity surface; $P_p = 0.5$ according to condition (15a); $x_g \approx l_m / \lambda_{\max}$ according to condition (11). In this relationship, the dependence on n is considerable only for the first large vortices ($x \leq x_{p1}$); during subsequent formations of large vortices from the traces of breakdown the value of n is close to unity by virtue of sufficient uniformity of the composition in the trace ($\rho_1 \approx \rho_2 \approx \rho_{cp}$).

$$\bar{l}_n(x, y, z, t) = \frac{1}{N} \sum_{i=1}^N l_i(x, y, z, t) \quad (8)$$

We will dwell now on the physical interpretation of the parameters of a diffusion model from the viewpoint of the vortex model. The scale of the width of the profile of concentration is not changed from the breakdown of a large vortex and is equal to the scale of large vortices, so that always $e \approx \bar{l}$. The scale of the field of longitudinal velocities changes with the breakdown of the vortex from scale \bar{l} to scale $l_p \ll \bar{l}$ (l_p - the scale of vortices in the trace of breakdown, it is the scale of the thickness of the new "discontinuity surface"), so that

*[Translator's note: cp = average].

$$z_0 \approx l P_0 - l_0 P_0 \approx l_1 P_0 \quad \text{with } l_1 \approx l$$

$$\text{and } \rho_0 \approx \rho_1^2 - 2 P_0 P_1 \frac{l_0}{l_1} \approx 0.5 \quad \text{with } l_1 \approx l_0 \quad (17)$$

The degree of uniformity of mixing N is not the same, either: within the limit ($\tau_{0.p}/\tau_B \ll 1$ and $l_p \ll l$) it is close to zero and unity respectively in a large vortex and trace of breakdown (see Fig 7b), so that

$$\bar{N} = N_1 P_1 + N_2 P_2 \approx P_1 N_1 \approx 0.5 \quad \text{with } N_2 \approx 1. \quad (18)$$

Let us The average shift of the field of concentrations along the of the y-axis (parameter a_T) is explained by the average stagnation of the volumes of a more rapid flow; the shift of the field of longitudinal velocities (parameter a_V) is caused by the fact that the centers of vortices y_c and the sections of discontinuity do not lie on the x-axis; their mean position is determined by the integral law of conservation of the impulse flow.

From (12) by the usual method we determine the connection between the In conclusion, let us illustrate the analytical possibilities of the model with its agreement with the experiment. The field of average velocities is determined by the relationship

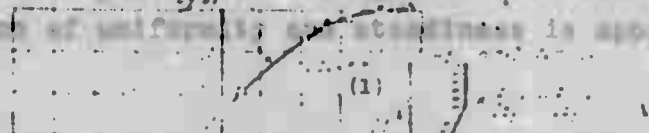
$$u = P_0 \int_0^{\infty} \bar{u}_0 \frac{1}{l} p(l) dl + P_1 \int_0^{\infty} \bar{u}_1 \frac{l_0}{l_1} p(l_1) dl_1, \quad (19)$$

where $E(x_1, k_3, \omega)$ we still considered the average of the ensemble of realizations of turbulent boundary layers on the plates:

$$p(l) dl = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \delta(l - l_i) = \frac{1}{2l} \frac{d}{dl} \left(l^2 \frac{d}{dl} \right) p(l);$$

$N = \int_0^{\infty} p(l) dl$ is the overall number of vortices which pass through cross section x during infinitely large time t ; $\bar{l} = \int_0^{\infty} l p(l) dl$; $\frac{\bar{l}}{l} = \frac{3l_0}{2l_1}$ - pressure $P_0(x_1, l_0, \omega)$. If we assume that section x is at (x_1, k_3, ω) the time (relative) of the passage of a vortex with dimension $2l$ through the cross section x ; $\bar{u}_0(y - y_c)$ - the profile of the

longitudinal average velocity in a large vortex; u_p - in the trace of breakdown. ... difference of the investigated points. In a developed turbulent boundary layer on a smooth plate with zero gradient of mean pressure, the condition of uniformity of turbulence is approximately satisfied.



for the calculation of first order (2) according to formula (11) it is necessary to know the analytical form of spectrum

$E_{v_1 v_1}(x_1, x_2, x_3, k_3, \omega)$. The most detailed results of the

correlation function **Fig. 8. Layer of mixing: experimental velocity measurements of Zhestkov and Albertson** $(2a_0 = 25.4 \text{ mm}; v_2 = 44 \text{ m/s}; \sigma_v = 7\%)$, but

even these results are ≈ 0.75 in order to construct the correlation function **KEY: (1) Calculation; (2) Velocity in its distribution in the vortex.**

immediate **Figure 8 presents the results of the calculation of the free layer of mixing and the experimental points [15].** **Figure 9 gives the experimental profile of the average velocity at the wall [10] and the simplest calculated version of the model (absence of the slippage of the vortex with the maximum slippage of the trace of breakdown)**

$$E_{v_1 v_1}(x_1, x_2, x_3, k_3, \omega) = \frac{1}{2} \left[\frac{v_1^2(x_1)}{v_1^2(x_2)} + \frac{v_1^2(x_2)}{v_1^2(x_1)} \right] \left[1 - \Phi\left(\frac{y}{z_p}\right) \right] + 1 - e^{-\frac{v_1^2}{v_2^2}} + \dots \quad (12)$$

For the intensity of the longitudinal component the first term of the Taylor series $\left[1 - \Phi\left(\frac{y}{z_p}\right) \right] + 1 - e^{-\frac{v_1^2}{v_2^2}}$ of the wall takes the form $(x = 76 \text{ cm}, u_0 = 1280 \text{ cm/s}; z_p = 0.605 \text{ mm}, z_1 = 6.3 \text{ mm}, P_p = 0.67)$.

$$T = \frac{V_p^2(x)}{a^2} \quad (13)$$

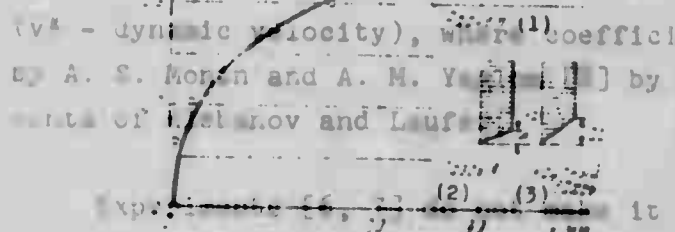


Fig. 9. Turbulent boundary layer; A. A. Townsend's experiment. Results of the measurement of Monin and Yaglom and Townsend.
KEY: (1) Calculation; (2) Discontinuity; (3) Large vortex.

it possible to predict changes in $E_{v_1 v_1}(x_1, x_2, x_3, k_3, \omega)$ depending on x_2 and x_3 . However, it

Consideration of the discontinuity (more precisely, the laminar sublayers on the lower and upper boundaries of the vortex and trace of breakdown) provides profile \bar{u} with the characteristic inflection point which disappears with an increase in the dispersion of scales l for x .

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By $v_1 v_1 (x_1, x_2, x_3, \tau)$ we will understand the space-time spectrum taken for the smallest distances from the surface of the plate. We take the correlation factor $r_{v_1 v_1} (x_1, x_2, x_3, \tau)$ in the form

$$r_{v_1 v_1} (x_1, x_2, x_3, \tau) = \exp[-c(|x_1 - x_2| - 0.1|x_3| - 0.1|\tau|)] \quad (15)$$

which corresponds the model of the boundary layer which considers the degradation of turbulent vortices in the process of their transfer at a velocity v_0 to $0.5v_0$ in direction of average movement. From a comparison with Favre's experimental curves it is possible to determine values c , β and γ :

$$c = \frac{2.5}{v_0}, \quad \beta = \frac{1}{v_0^2}, \quad \gamma = 0.1$$

The spectrum of pressure on the surface of the plate is obtained by substituting expressions (12)-(14) into the right portion of (11):

$$P_{xx}(x_1, x_2, x_3, \tau) = \frac{1}{2} \rho v_0^2 \tau_w^2 \left(\frac{1}{\tau_w} + \frac{1}{\tau_w} \right) \quad (16)$$

where $\tau_w = \rho v_0^2$ is the stress of friction on the wall.

From (15) it follows that for the mean square pressure on the wall the relationship is correct

$$p^2 = 4\sigma^2 \gamma^2, \text{ and } \gamma = \frac{1}{2.61} \quad (17)$$

Substituting the values of α and γ determined above, we obtain:

STUDIES OF TURBULENT CONFINED AND OPEN FLOWS ACCOMPLISHED IN THE INSTITUTE OF HYDROMECHANICS OF THE ACADEMY OF SCIENCES OF THE UKRAINIAN SSR sions of Kraichnan [1] about the law of the square pressure to the stress of friction on the wall and with the estimates of the proportionality factor carried out by Lilley [2]
I. L. Rozovski
(Kiev)

$$(1.7 < \overline{p^2}/\tau_w < 3.1)$$

Review

From (17) it is apparent that the proportionality factor γ determined by the scale of slope of the profile of intensity of the longitudinal component of the fluctuating velocity to the plane of the wall and by the ratio of correlation of this same component. The experiments of Willmarth and Wooldridge [9] give values of this coefficient of 2.7 and 2.2 for two different conditions. In connection with the fact that at the present time in the USSR, unfortunately, there is no sufficient practice in the reliable use of instruments of the hot-wire anemometer type for dropping liquids, during the studies the following were basically used: 1) the method of visual study of a flow by means of the introduction, into the flow, of solid or liquid particles - indicators with cinematographic photography of the flow and subsequent statistical processing of the photographs (subsequently, we will use for it the shortened name: photographic and cinema method); 2) one- and two-component velocity sensors based on the principle of the dynamic effect of a flow on bodies introduced into it.

The photographic method was developed in the USSR by M. A. Velikanov, N. P. Zrellov, B. A. Fidman and I. K. Nikitin. I. K. Nikitin [1] proposed using solid particles as indicators - balls from a mixture of paraffin with whiting that has identical volume weight to water, and defatted fine aluminum powder with particle dimensions of 10-100 μ . Illumination during photography is accomplished at specific time intervals by a flash bulb with power supply from a pulse generator. With photographing in mutually perpendicular planes, it is possible to obtain a three-dimensional picture of the flow.

In order to determine possible errors with the photographic method with the use of aluminum powder as the indicators, E. V. Zalutsky accomplished an analysis of the structure of turbulent flows with a shift by the following scheme. Following Townsend [2], the turbulent flow was considered as the superposition, on the main flow, of simplest vortex structures of different scales and intensities. Using Laufer's data [3] on the energy spectrum of a turbulent flow with a shift, it was possible to obtain the representation of scales and velocities of different vortices. Then, having made use of Favre's works [4], it was possible to determine the possible deviation of the trajectories of solid particles of different dimensions from the trajectories of liquid particles.

The analysis showed that for conditions under which laboratory investigations of water flows with velocities up to 1 m/s and linear dimensions (depths) on the order of 0.3-0.5 m (which corresponds to Reynolds numbers on the order of $1 \cdot 10^5$ - $5 \cdot 10^5$) are usually accomplished, with the use of aluminum powder it is possible to catch vortex structures which also approach turbulence for size and microscale.

The advantages of the visual method are the absence of noticeable distortions of flow, simplicity of equipment, the possibility

of measurements in flows with a high intensity of turbulence, and also the possibility of making measurements in the immediate proximity of the walls. The main deficiency of the method - the laborious processing of the results. The question of the automation of the processing of the results of the photographic method and direct input of information into a computer thus far does not have a satisfactory solution and it should be the subject of further research. In the case of the use of equation (1), the pressure characteristics can be refined if fine measurements of the moments of the second order of the dynamic effect of flow is

One of the instruments based on the dynamic effect of flow is a ball two-component velocity sensor, investigated in detail in the Institute of Hydromechanics of the AS of the Ukrainian SSR by

B. M. Yegidis [5]. A diagram of the instrument, which makes it possible to simultaneously measure two velocity components at one point, is shown in Fig. 1. Investigations showed that the force which acts on the ball in a nonstationary turbulent flow differs from that in a corresponding stationary flow as a result of: a)

the virtual mass effect and b) the phenomenon of "hydromechanical inertia" which possesses a vortex zone behind the ball. Because of the latter circumstance, with the acceleration of the flow the resistance coefficient of the ball is relatively less, and with deceleration - greater than in a stationary flow.

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Fig. 1. Diagram of a two-component sensor. 1, 2 - mutually perpendicular plates with wire strain gauges glued on them; 3 - balls which perceive the dynamic pressure of the flow.

Furthermore, as a result of some instability of the position of the line of separation of the boundary-layer, behind the sphere the fluctuation of the drag force is observed (and when a gradient of average speed is present - also lift) even in a nonturbulent external flow.

The intensity of these apparent fluctuations of velocity being recorded by the instrument attains 5-7% of the forward velocity. Therefore, the ball sensor is recommended for use only for studying comparatively large vortex structures in flows with a high intensity of turbulence where the errors indicated above will play a comparatively small role.

1. Studies of uniform flows in water conduits with smooth and rough walls. One of the most important in practice is the study of a steady flow in pipes and open ducts of constant cross section uniform in the direction of main flow (uniform flow). The distribution over the cross section of the averaged (for time) velocities (Kiev) for the simplest flows of such a type (circular pipe, wide rectangular delivery duct) has been studied in sufficient detail, and the turbulent structure - considerably less. The velocity field near a solid wall is also insufficiently investigated, especially in the case of rough walls. Turbulence in unsteady flows in comparison with steady flows even with comparatively small local acceleration. The detailed investigations in confined and open ducts of lead to rectangular cross section were executed under the guidance of flows. I. K. Nikitin over a wide range of change in the Reynolds numbers and roughness of the walls. These studies made it possible to obtain the distribution of average velocities, single-point moments coefficients for the components of the fluctuations of velocity and, in a number of cases - also two-point correlations and one-dimensional energy spectra of the fluctuations of velocity.

The purpose of this work is the creation of a method of calculating for the processing and theoretical explanation of the results of the experiments, I. K. Nikitin proposed using a linear report parameter - the thickness of the wall layer δ , with aid of which he managed to obtain a "two-layered" distribution curve of velocities, general for the cases of smooth, "slightly rough," and completely rough walls, and a general law of resistance for flows in all cases. The physical conception of I. K. Nikitin consists in the following: In rough channels, similar to smooth ones, there is a change in the turbulent structure as a result of accelerations, also

differential equations for the values which characterize
 is a region, in which the specific properties of a flow which.
 such equations for the second moments are used in the
 flows around the projections of roughness are exhibited. The
 theory proposed by A. N. Kolmogorov and I. K. Nikitin
 unique structure of vortices, caused by the detached flow around
 and developed in the works of A. N. Monin
 the roughness elements, creates specific additional "viscosity"
 and other authors. This theory is accepted for the
 whose effect disappears only at a specific distance from the
 of a closed system of equations on the basis of
 apexes of the projections of roughness. Furthermore, if we perform
 Reynolds, and the continuity and balance of longitudinal
 a three-dimensional averaging of the longitudinal velocity compo-
 nents between the projections of roughness, then the distribution
 restricting ourselves to an approximation of the
 along the vertical of such "averaged velocities" turns out to be
 smoothly changing motion. It is possible to simplify the
 similar to the linear (analogy with linear velocity distribution
 equations. The equation of motion derived from Reynolds' equations,
 in a viscous sublayer).
 after estimates of the order of terms [4] similar to the way

in which this is done for the boundary layer, take the form
 The presented considerations allowed I. K. Nikitin to propose
 the following formula; for the distribution of average velocities
 along the vertical of a plane turbulent flow). (1)

where ψ is the piezometric function, $\psi = 5.0 \left[1.13 \ln \frac{y}{\delta} + 1.5 - 0.5 \frac{y}{\delta} \right]$, ψ varies in the (1.1)
 longitudinal and transverse directions; U_1, U_2 - projections of
 the vector \vec{U} of the averaged velocity at the given point; u_0 - dynamic
 velocity; y - distance from the wall; δ - the thickness of the
 wall layer, for a smooth wall equal to the thickness of the viscous
 sublayer and for a completely rough wall - equal to the average
 height of the projections of roughness; the characteristic
 longitudinal and transverse dimensions of flow) which is used in
 the simplification of the equations of motion.
 For a slightly rough channel, the boundary of the wall layer
 passes higher than the projections of roughness. The curve pro-
 vides a smooth coupling with the straight line of velocity dis-
 tribution in the sublayer. (This is discussed in greater detail
 in the report of I. K. Nikitin contained in this collection.)
 Some deficiency in the curve is its "two-layer quality" and
 velocity gradient different from zero on the axis of flow (at the
 free surface of an open duct). (2)

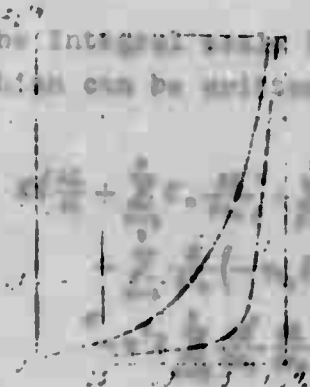
The feature of equation (2) is its agreement with the data of numerous experi-
 ments of turbulence in the Institute of Hydromechanics of the Academy

of Sciences of the Ukrainian SSR was shown by the curve of Pai Shih-1 [6], suitable for the entire flow in a smooth channel including the viscous sublayer (single-layer scheme). However, unfortunately, the coefficients which enter this formula depend on the Reynolds number in a sufficiently complex manner.

In the second place, from having a sufficiently precise distribution curve of average velocities and knowing the law of resistance, it is possible to calculate the Reynolds stress of friction $\overline{u_1 u_2}$ at any point according to the depth of the plane flow and to completely examine the energy balance of averaged motion. In particular, it is possible to find the depth distribution of the rate of the generation of the energy of turbulence and energy losses of the averaged motion for different Reynolds numbers and walls of different roughness [7]. The results of one such calculation are shown on Fig. 2, on which the distance from the wall is laid off along the ordinate referred to the half-width h of the channel. It is interesting to note that in a smooth channel the generation of turbulence and viscous dissipation (i.e., what in hydraulics is customarily called the energy losses) is concentrated in a relatively thin region at the wall, while for rough walls this region embraces a considerably greater part of the flow, and is confirmed by test data [7], we obtain for the turbulent friction

$(\overline{u_1 u_2}) = -L \overline{v^2}$ As a result of the generalization of the experimental data, I. K. Nikitin proposed empirical formulas for determining the distribution in the plane flow of longitudinal and vertical mean square fluctuating velocities $\overline{u^2}$ and $\overline{v^2}$, referred to u_* [1]. With the aid of these formulas it is possible, with some degree of approximation, to find the distribution along the vertical of the averaged value of the kinetic energy of turbulence \overline{q} . If we, further take the well-known expression $\nu_t = \overline{q} / \overline{v^2}$ for the coefficient of turbulent viscosity, then hence it is possible to calculate the distribution for the cross section of scale l . Such an analysis was performed by Ye. V. Yeremenko (see the report contained in this collection).

For the integral curve, Fig. [2], integral curve of the equation which can be used in the form:



of the distribution for depth of the total losses of energy of averaged motion (in percentages of the full magnitude of losses) in plane flows with smooth (1) and rough (2) bottom; $Re = 20,000$. (3)

If we make an estimate of the order of the terms of equation (3), then the sufficiently precise knowledge of the law of resistance and distribution of velocity and the kinetic energy of turbulence makes it possible to calculate with some approximation the distribution over the cross section (excluding the bottom region) of the rate of dissipation of energy of turbulence ϵ and correspondingly the value of the scale of dissipation λ [8]. For this it is possible to use the formula

the unsteady motion scale L can also be considered independent of coordinate x_1 and time t , i.e., can be accepted in the calculations

The experimental value of coefficient C_1 was found by applying the equation of balance of fluctuating energy to that point of flow in which, from rough estimates, the diffusion of the energy of turbulence is absent and the dissipation of turbulent energy is equal to its generation. Thus the value $C_1 = 0.077$ was obtained sufficiently close to those found by other authors.

With the use of such calculations the distribution of the amount of velocity of dissipation was obtained for the cross section and then the values of the scale of the dissipation λ_A and the Kolmogorov scale η were found.

In calculations of L for U , the formula of I. K. Mikitin was accepted for a two-layered model which makes it possible to calculate in practice the velocity u to the viscous layer. This formula takes the form:

is given in Fig. 3 (here h is the half-width of the duct, u_* - dynamic velocity). The comparison of the calculations with the direct measurements of the scale of dissipation made by D. Laufer in a flat duct with smooth walls gave satisfactory agreement.

where δ is the thickness of the boundary layer determined from relationship $\delta/\lambda = 0.42$ is the parameter reflecting the Reynolds number effect. For determining the scale of turbulence also obtained according to I. K. Nikitin's data for the less Kolmogorov scale η' , equality $\langle u^2 \rangle = \langle u'^2 \rangle$, is being formula:

Fig. 3. Graph of dimensionless Kolmogorov scale η' .

$$K = \frac{1}{2} \left[\left(0.3 + \frac{1}{\delta} \left(\frac{1}{\delta} - 1 \right) \right)^2 + \left(\frac{1}{\delta} - 1 \right)^2 \right]^{1/2} \quad (6)$$

I. K. Nikitin used the data on the velocity distribution and a value of the friction drag in a uniform flow obtained by him for the further calculation of the turbulent boundary layer on a flat plate further with a rough surface, and also for the calculation of the boundary-part L layer and heat-mass transfer through a free water surface covered theory with waves [9, 10]. It means that in the region $x_2/h \geq 1$, where the energy of turbulence is finite, the dissipation of energy $\epsilon = \epsilon_2^{1/2}$. Studies of nonuniform turbulent flows. If the structure of uniform turbulent flows and the laws of resistance in them are upper investigated sufficiently well at the present time, then nonuniform turbulent flows (to which, for example, pertain the flow in used diffusers and converging nozzles of pressure systems, flows with indicate curves of backwater effect and fall-offs in open ducts and others) are studied very little in this respect.

$$\frac{u^2}{u_*^2} = 1 + \frac{1}{2} \left(\frac{y}{h} \right)^2 + \frac{1}{2} \left(\frac{y}{h} \right)^4 \quad (7)$$

The studies of nonuniform flows in open chutes were accomplished in the Institute of Hydromechanics the AS of the USSR Ukrainian SSR by E. V. Zalutskiy with the aid of the photographic experiment method [11]. The experiments were preceded by an analysis of possible systematic errors connected with the use of this method for three-dimensional-nonuniform flows and the determination of

necessary corrections. The distribution of the averaged and fluctuating velocities in open flows which expand and narrow along the length under conditions of the flow close to plane-parallel was further investigated. The angle of divergence and convergence comprised approximately 2° and the value of the Reynolds number changed within limits of 5000-20,000. The results of experiments in the form of the energy-distribution curves of turbulence referred to the square of the average velocity for cross section are shown on Fig. 4.

The calculations according to this formula also confirmed the presence of self-similarity and led to the energy distribution of turbulence along the vertical with a nonuniform flow in smooth (a) and rough (b) channels (a - the height of projection of roughness). The results of the different determination of the integral scale L (see Fig. 1) that it practically coincides with the relationship constructed with the use of formulas (5)-(7). Moreover, Fig. 1 gives G. S. Glushko's relationship (1), also recalculated by means of division by ϵ equal to 5. It coincides with the obtained relationship of average velocities becomes more complete, and with deceleration completeness of the diagrams decreases. However, detailed photography of the distribution curve of velocities close to the bottom showed that the velocity gradient expressed in the corresponding dimensionless quantities not only does not increase with averaged velocity. A similar circumstance, apparently, also takes place with scale L . The accomplished calculations and provide a certain confidence in the correctness of the obtained relationship $L/h = f(x_2/h)$ (curve 5 on Fig. 1) (Fig. 4). In the wall region, no considerable difference between fluctuating velocities u' and v' in the cases of uniform and nonuniform flows was observed; the flow rapidly "adapted itself" to new conditions. In the nucleus of the flow, with the acceleration of flow a pronounced decrease in the level of turbulence is very small is examined, it can be assumed that extrapolation will not lead to noticeable inaccuracies.

(relations 1 and 1) was observed in comparison with the level for a uniform flow, and with deceleration - an increase in this value.

Let us dwell on the approximation of the individual term of the ratio. According to data of the investigations, the energy balance of the averaged motion in a nonuniform flow was calculated and the coefficients of resistance C_f and energy losses λ were determined. Let us note that in a nonuniform flow, unlike a uniform flow, these values do not coincide basically because of the gradient of normal turbulent (Reynolds) stresses. (8)

The results of the calculations led to somewhat paradoxical conclusions. The coefficients of resistance C_f and energy losses λ both in a decelerating as well as in an accelerating flow prove to be less than under corresponding conditions in a uniform flow. This conclusion correlates with the data presented above about the reduction in the velocity gradient close to the bottom in an accelerating flow. Physically this phenomenon can be explained by the fact that in an accelerating flow, where the average level of turbulence for the cross section is considerably lower than in a uniform flow, the main source of hydraulic losses - the generation of turbulence $\overline{u_1 u_2} (d\overline{u_1}/dx_2)$ - decreases due to a decrease in $\overline{u_1 u_2}$ and in a decelerating flow - decreases due to a decrease in the velocity gradient $d\overline{u_1}/dx_2$ in that part of the flow, where $\overline{u_1 u_2}$ has a significant value.

to the binomial formula of Hotta [1] [2] obtained for uniform turbulence. The presented considerations are purely qualitative. The studies of these interesting cases of flow in the Institute of Hydromechanics of the AS of the Ukrainian SSR continue.

of the approximating expression for terms which express the turbulence. 3. Studies of nonstationary turbulent flows in delivery pipes and open ducts. For the solution of a number of practical problems, of great interest are nonstationary turbulent flows in which the average statistical characteristics of the flow are a function of time (random). Examples of such problems are unsteady motion in open channels and pressure systems and calculations where $\overline{u_1 u_2} = \overline{u_1} \overline{u_2}$, $\overline{u_1 u_2}$ is constant.

of a turbulent boundary layer and resistances during the unsteady motion of a body in a liquid.

First of all, it is necessary to determine the possible methods of studying such flows. If the time scale of change in the averaged characteristics of a turbulent flow is considerably greater than the corresponding characteristic scale of fluctuations of velocity, then the averaging operation does not cause difficulties. Different approximate methods of the analysis of non-stationary random functions were proposed by A. N. Patrashev [12] and V. S. Pugachev [13].

As is known, the only strict method of obtaining the characteristics of turbulent flow in the most general case is the method of statistical averaging or averaging on ensemble. In the works of the Institute of Hydromechanics of the AS of the Ukrainian SSR an experimental method which corresponds in theory to statistical averaging was also developed. The experiment was repeated many times under invariable initial and boundary conditions and, in a certain phase of flow (time coordinate) photography of the flow was conducted with a sufficiently small exposure. Thus, for each time coordinate it was possible to obtain a sufficiently long statistical series of the values of the interesting quantities.

Detailed studies of three cases of flow were accomplished by the described method: 1) discontinuous wave in an open flow, 2) smoothly changing unsteady stream in an open flow 3) unsteady motion in a delivery pipe,

(12)

Extremely curious results were obtained by Ye. V. Yeremenko for during the study of the discontinuous wave which is formed in an open duct with the sudden opening of the gate [14]. Apropos the structure of such a wave, different opinions have been voiced in the literature: 1) the flow is accomplished according to a pattern of two layers, whereupon the upper layer moves considerably obtain the following expression for the diffusion of pressure energy in the form of a series:

faster than the lower with the formation of an interface between them, 2) the surface layer leaking in "extrudes" the liquid ahead of itself and forces the flow to move accelerated along its entire depth.

The results of the detailed experimental studies of this case of flow in the form of curves of the change of the components of average velocity and moments of fluctuating velocities for different cross sections of flow are shown on Fig. 5. As is evident, in actuality the flow according to the "water on water" diagram is very clearly realized, whereupon the separation boundary is the intense source of powerful fluctuations of velocity. Therefore, one ought, in essence, to approach the analysis of such motion from the positions of the theory of free turbulent flows, which up to now, has not been done.

The studies of a smoothly changing unsteady turbulent flow in an open channel by the method described above were performed by A. N. Shabrin [15]. The results of these studies in some respects are analogous to the results given above for nonuniform motion. It was precisely here that the phenomenon of the "inertia of turbulence" was observed, i.e., the lagging of a change in the level of turbulence behind a change in the velocities of the averaged flow. The diagram of the averaged velocities changed comparatively little.

The unsteady flow in open channels is distinguished by great complexity: both local and convection accelerations take place, in which regard, often of different signs. Furthermore, in the process of flow the flow geometry (free surface) changes. In order to investigate this phenomenon in a purer form, it was decided to pose the experimental studies of unsteady flows which are accelerated and decelerated with time in a rectangular delivery duct under conditions close to the conditions of a plane flow.

Introducing expressions (11), (12), (13), and (14) into equations (1) and (2), considering that these expressions and coefficients γ , δ , ϵ , ζ are universal solving the continuity equation and relationship $\rho = \rho(x_1, t)$ represented in Fig. 5, we will obtain a closed system for the calculation of the velocity motion of the particles in the

Introducing the initial conditions (represented by the lines on top)

(g_0 - the projection of the velocity on the x_1 axis, U_0 - average velocity, γ - the transverse component of the velocity, δ - the gyration number, ϵ - the necessary correction, ζ - the form

(lines above the diagram represent quantities are constant). Coefficient $k = h_0$ is a function of the local coordinate x_1 and time t .

The system of equations (15) can be solved numerically with the aid of the local one-dimensional method A. A. Gerasimov [14] for equations of the form

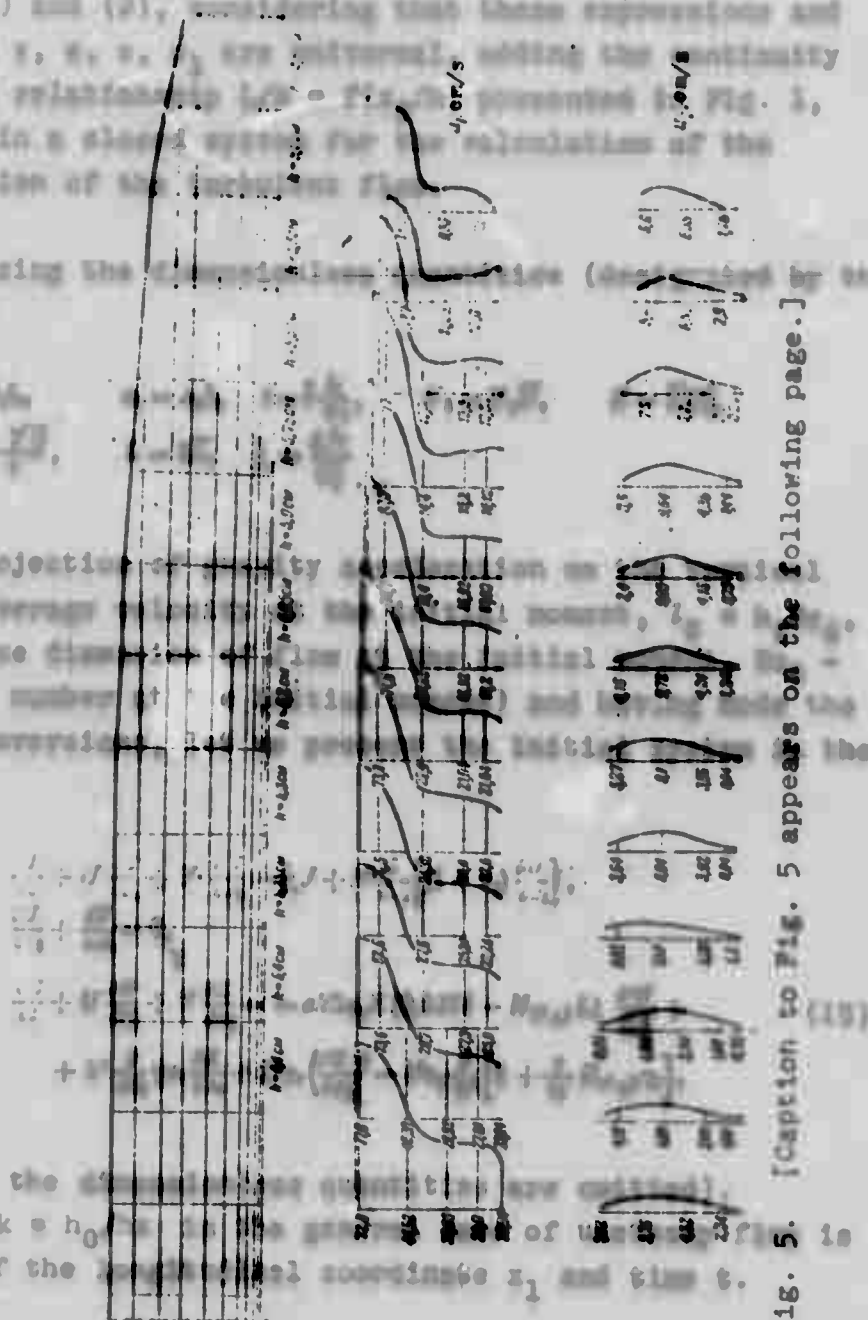


Fig. 5. [Caption to Fig. 5 appears on the following page.]

Both the equations of motion and the equation of energy of the system (15) possess the same feature: the presence of the term $\frac{1}{2} \frac{d}{dt} \left(\frac{1}{2} \frac{d}{dt} \right)$ in the approximation of the initial equations by the virtue of the initial conditions (15) the part of the unknown function according to the previous then we know that the system (15) is formulated by means of the differential equations, consisting of partial differential equations of the second order of one-dimensional case is indicated.

We will restrict ourselves to an one-dimensional case, i.e., we will consider the problem of the unsteady motion of a pressure plane in a system of equilibrium of a fluid. The problem of unsteady motion of a pressure plane in a system of equilibrium of a fluid is the problem of unsteady motion of a pressure plane in a system of equilibrium of a fluid.

The straight line on which the pressure plane is placed is the wall to the fluid. The fluid is divided into two parts by the wall, and the partial derivatives are replaced by finite-differences expressions:



Fig. 3. Distribution of kinematic characteristics in a discontinuous wave (u_1, u_2 - the longitudinal and vertical components of averaged velocity, u_1', u_2' - corresponding fluctuation in velocities).

(16)

Such studies are being accomplished in the Institute of Hydro-mechanics of the AS of the Ukrainian SSR by S. B. Markov. They are still unfinished, but some results have already been obtained.

The studies were accomplished in a tube with a cross section of 10×38 cm. The flow rates changed within limits of from 0.16 to 0.72 m/s with acceleration on the order of 0.1 m/s^2 ; the maximum value of Reynolds number was 72,000. The results of the experiments showed that with acceleration of the flow in the mass of the flow the completeness of the velocity curve increases. The reverse picture is observed during the deceleration of the flow. The measurement of fluctuating velocities also showed in this case the lagging of the change in intensity of fluctuations behind a change in the field of averaged flow rates.

(17)
4. Approaches to calculations of nonuniform and unsteady turbulent flows. A semiempirical method with the use of a system of Reynolds equations and the equation of the balance of turbulent energy was used for the calculation of the field of averaged and fluctuating velocities in nonuniform and unsteady flows. This method was successfully used by A. S. Monin, I. Rotta, G. S. Glushko, and V. B. Levin for the calculation of uniform flows and flows in the turbulent boundary layer. As is known, the basic idea of the method consists of the closure of a system of equations by using a series of approximate dependences for determining turbulent viscosity and the dissipation of energy and diffusion terms in the equation for the balance of energy. Such basic approximating dependences are:

a) the expression given above for the coefficient of turbulent viscosity:

$$\text{coefficient } \mu_t = \rho \cdot \nu_t = \rho \cdot \frac{u'^2}{N} \quad (4.1)$$

b) the expression for the dissipation of the energy of turbulence:

$$\varepsilon = \nu \frac{\partial^2 \bar{q}}{\partial x^2} + C_1 \frac{\bar{q}^2}{T} \quad (4.2)$$

where the addend corresponds to small Reynolds numbers, and the augend - large; with sufficiently large Reynolds numbers the addend can be disregarded and then we will obtain expression (1.2);

c) the expression for the diffusion of the energy of turbulence; considering that transfer of the energy of turbulence in a transverse direction is gradient type diffusion, it is possible to express the diffusion terms by the dependence

$$D_1 \frac{\partial^2 \bar{q}}{\partial x^2} = \eta_1 \frac{\partial \bar{q}}{\partial x} \quad (4.3)$$

(17)

Furthermore, it is necessary to have an expression for scale l . As is known, G. S. Glushko [16], on the strength of a series of experimental data, obtained for l a sufficiently complex dependence (graphically it is a broken line).

Thus, a closed system of equations is obtained with some empirical coefficients for finding the components of averaged velocity and the kinetic energy of turbulence. This procedure was improved somewhat by Ye. V. Yeremenko and is used for the calculation of smoothly changing motions in wide open and delivery ducts, the flow in which is close to plane.

Using the experimental data of I. K. Nikitin and the curve of Pai Shih-I which provide good agreements with the experimental data close to the axis of a plane confined flow or close to the free surface of an open flow, the distribution of scale l over the cross section was refined.

Analysis showed that with sufficiently large Reynolds numbers the distribution of scale l stops depending on this number.

Furthermore, unlike the data of G. S. Glushko, on approaching the

axis of flow (or the free surface of an open flow) the value of λ does not decrease, but it increases monotonically and it approaches a constant value.

A substantial improvement of the method was made with consideration of the diffusion of the energy of fluctuations of pressure. As is known, the fluctuation of pressure at a given point of flow is determined by the velocity field in the entire flow and is expressed through this field by the known adiabatic equation. Consequently, the diffusion of the kinetic energy of turbulence and the diffusion of the energy of the pressure fluctuations are different phenomena. The data of D. Laufer and I. K. Nikitin show that the nature of a change in these values over a cross section is completely different, and for order of magnitude they are close to each other; therefore, neglect of the diffusion of pressure energy in general is not justified.

Ye. V. Yeremenko, using the equation which connects the fluctuations of pressure and velocity, obtained an expression for the value of the diffusion of energy through the gradient of averaged velocity and distribution of scales over the cross section and composed a closed system of equations which makes it possible to perform calculations of nonuniform and unsteady turbulent flows. The calculations were carried out to the end for the case of a pressure plane flow and satisfactory agreement with the experiments of S. B. Markov was obtained (see the report of Ye. V. Yeremenko contained in this collection).

It appears that this method can be used successfully for the calculations of unsteady and nonuniform open flows and, in particular, it will make it possible to obtain sufficiently precise data on resistances with these forms of motion.

5. Studies of the structure of turbulent flows with interfaces. In connection with the solution of the practical problem -

the expansion of the flow which issues from the opening of a structure; - with the aid of the photographic method and of a dynamic sensor the structure of such a flow was investigated in detail in the area of the interface between a forward moving jet and the closed vortex regions surrounding it. This case differs from the usual turbulent jet by the effect of the bottom and the walls which restrict the flow (the three-dimensional problem). in such a way that with certain constants the static problem was solved. As a result of the studies, a large amount of experimental material has been accumulated about the values of fluctuating velocities and their single-point covariances [17]. The experiments showed an increase in the intensity of turbulence in proportion to the flow expansion. The maximum of turbulent tangential stress is noted immediately close to the interface where the greatest gradient of the averaged velocities and the maximum value of the velocity fluctuations occur. The interfaces between the forward moving liquid and the vortex zones are a powerful source of turbulence of flow. Great fluctuations of velocity and pressure cause considerable dynamic loads on the structures enclosing the flow, and they also sharply increase the capability of the flow to wash out the ground behind the structure. [7], in which regard it was assumed that $Re_F = 22.0$.

On the basis of experimental curves of fluctuations of velocity recorded by the sensor, the autocorrelation functions of the fluctuations of velocity were constructed and by means of their transformation - frequency spectra. The autocorrelation function in the region of the interface close to the opening has the expressed character of a sinusoid with attenuated amplitude and only at a sufficient distance from inlets takes the form characteristic for the correlation of the random variables.

The results of the calculations are given in the form of graphs in Fig. 2. In accordance with the aforesaid (about correlation functions, the spectral curve (dependence of amplitude on frequency) directly behind the opening has a pronounced maximum in a specific frequency region, which tells of the periodic separation of the vortices with uniform motion (curves 1). With decelerated motion (curves 3)

along the entire depth the curve becomes less complete. These results (Karman street type). In proportion to the distance from the opening, the maximums are alleviated, moving into the region of lower frequencies, and the spectral curve takes a form usual for turbulent flows. This tells about the gradual transition from regular perturbations to a purely random turbulent structure.

The energy of turbulence presented in Fig. 2 in the form of graphs of relative values ϵ/ϵ_0 .
6. Studies of turbulent flows which carry solid particles. For a number of years, in the Institute of Hydromechanics of the AS of the Ukrainian SSR experimental studies were conducted of the kinematic structure and dynamic characteristics of flows which carry suspended solid particles in comparatively large quantities and (from 3 to 25% by volume). In the relative value of the energy of turbulence in comparison with uniform motion. These results are also in qualitative agreement with the data of S. B. Markov.

The studies were conducted over a wide range of change in the basic parameters which characterize the flow. The diameters of conduits changed from 100 to 900 mm, the flow rate -- from the minimum to 8 m/s, the size of particles -- from 0.1 to 40 mm, and the density of solid material -- from 1.4 to 4.5 t/m³. Sand, coal, gravel, waste products from iron-ore combines, etc., were used as the material being transported. Very great experimental material has been accumulated which still requires theoretical interpretation. We will dwell here very briefly on some of the results of the unsteady flows. It is now necessary, in the calculation of uniform motion, to check the universality of constants for different types of flows.

Highly-saturated suspension-carrying flows are characterized by great nonuniformity in the distribution of the concentration of solid particles along the vertical. The degree of this non-uniformity increases with a decrease in the average velocity and with an increase in the size and density of the particles.

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geometrical axis of the flow. With identical average velocity (identical flow rate of the liquid phase) the maximum velocity and the velocity gradients along the vertical become greater than for pure water. Thus, the solid components significantly affect the kinematic flow structure.

The resistance to the motion of a suspension-carrying flow in all cases was considerably greater than for pure water and, as a rule, it increased with an increase in the concentration. However, the latter was observed to a definite limit. With velocities of motion similar to the so-called critical velocities, at which the deposition of particles begins, resistance can decrease with an increase in concentration and, besides, rather considerably. With an increase in the size of the particles the resistance increases, however, only to a definite limit, after which it stops, depending on size.

The measurement of the fluctuation motion characteristics of liquid and solid particles showed that the energy of turbulence of liquid particles in the middle part of the flow proves to be greater than in the flow of a homogeneous liquid, and only close to the lower wall of the pipe, in the region of an extremely considerable concentration of solid particles, is a decrease in the energy of turbulence observed. On the average, over the cross section the energy of turbulence not only does not decrease, as is frequently accepted, but it increases rather considerably. This is discussed in greater detail in the report of N. A. Silin and V. F. Ocheret'ko (see this collection).

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1. Applying the Navier-Stokes equations, the fluctuations of pressure is obtained:

$$-\frac{1}{\rho} \nabla^2 p = \sum \frac{\partial^2 u_j^2}{\partial x_j^2} - \sum \sum \frac{\partial^2 u_j u_k}{\partial x_j \partial x_k} + \sum \sum \frac{\partial^2 (u_j u_k + u_k u_j)}{\partial x_j \partial x_k}, \quad (1)$$

where U_j , U_k are the averaged values of the velocity component and u_j , u_k - the fluctuating velocity components; angular brackets $\langle \rangle$ are the sign of averaging for probability.

The right side of equation (1), called the *kissatio function*, is assumed to be known in the sense which is usually kept in mind when they speak about the space-time random function.

The boundary conditions for equation (1) in the case of a motionless flat impermeable boundary, obtained from the Navier-Stokes equations, takes the following form:

$$\frac{1}{\rho} \frac{\partial p}{\partial y} = \nu \frac{\partial^2 v}{\partial y^2}, \quad (2)$$

where v is the component of fluctuating velocity along the y-axis (the y-axis is directed along the normal to the wall).

On the free surface of the flow the fluctuation of pressure is taken as equal to zero:

$$p = 0$$

(3)

or is given in the same way as is usually done in the study of waves on the surface of the water.¹

In many instances, for example in slightly nonuniform flows on a smooth bottom, boundary condition (2) can be replaced by a simpler one - uniform:

$$\partial p / \partial y = 0.$$

(4)

With such a boundary condition equation (1) was investigated for the first time, apparently, by Kraichnan in 1956 [2].

The possibility of the replacement of condition (2) by relationship (4) can be substantiated, for example, by Townsend's estimates ([13], p. 275), according to which in the boundary layer

$$\langle (\partial p / \partial y)^2 \rangle|_{y=0} \sim 3 \cdot 10^{-4}, \quad \langle \partial p / \partial x \rangle^2|_{y=0} \sim 2 \cdot 10^{-4},$$

so that the fluctuation gradient of the pressure at the wall along the flow is considerably greater than across the flow.

If we assume that the turbulent perturbations at the wall are planar, then, using the theory developed in [4], we obtain:

¹The recording of the boundary condition taking into account the action of waves and also some other possible boundary conditions are given in [1].

$$\langle (\partial p / \partial x)^2 \rangle = \int_{-\infty}^{\infty} |C(\omega, \kappa) / \kappa|^2 S_{\tau_{\tau}}(\omega, \kappa) d\omega d\kappa,$$

$$\langle (\partial p / \partial y) \rangle = \int_{-\infty}^{\infty} \kappa^2 S_{\tau_{\tau}}(\omega, \kappa) d\omega d\kappa = \langle (\partial \tau_{\tau} / \partial x) \rangle,$$

where $C(\omega, \kappa)$ is some universal function of frequency ω and wave number κ ; $\tau_{\tau}(x, y)$ is the fluctuating component of tangential stress on the wall, and $S_{\tau_{\tau}}(\omega, \kappa) = \langle \tau_{\tau}^* \tau_{\tau} \rangle$ - the spectrum of the tangential stress on the wall (the asterisk above signifies a Fourier transform, the line above the variable - a complex conjugate value).

Using the empirical data on the spectrum of the tangential stress on the wall, it is possible to ascertain that $\langle (\partial p / \partial y)^2 \rangle_{y=0} < 1000 \langle (\partial p / \partial x)^2 \rangle$, and condition (4) becomes even more reliable.

2. In the plane flow with a free surface and slightly changing depth, the solution of equation (1) with boundary conditions (3) and (4) for points on the bottom of the flow has the following form [5]:

$$p(x, y) = - \int_{-1}^1 d\eta \int_{-\infty}^{\infty} G(\xi, \eta) / (x - \xi, \eta) d\xi. \quad (5)$$

The function of weight

$$G(\xi, \eta) = \frac{1}{2\pi} \ln \frac{\operatorname{ch} \frac{\xi^2}{2} + \sin \frac{\pi \eta}{2}}{\operatorname{ch} \frac{\xi^2}{2} - \sin \frac{\pi \eta}{2}} \quad (6)$$

decreases rapidly with withdrawal from the point of bottom in question ($y = -1$, $\eta = -1$; $x = x_0$, $\xi = 0$) into the depth of the flow ($0 > \eta > -1$), downward ($\xi > 0$) or upward ($\xi < 0$) along the flow (Fig. 1).¹

¹In formulas (5)-(7) the depth of flow is taken as equal to unity; axes y and η are directed upward, the reference point - at the surface of the water.

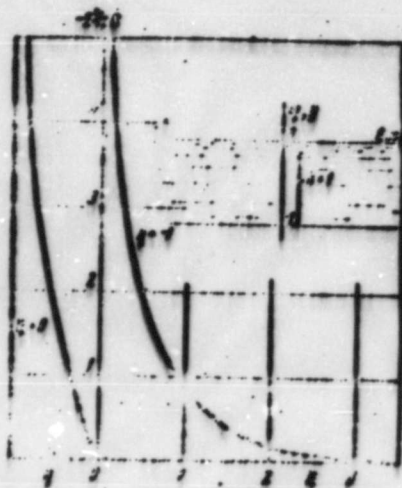


Fig. 1.

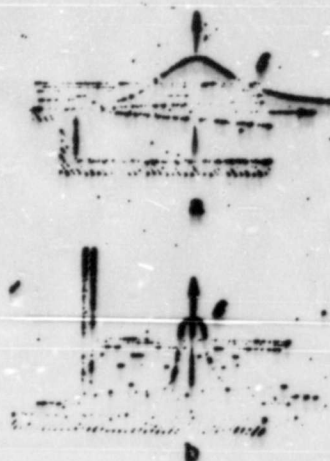


Fig. 2.

The longitudinal spectrum of the fluctuations of pressure on the bottom of a uniform plane flow can be simply calculated if the mutual longitudinal spectrum of the kinematic function in different layers of flow $S_p(\omega, n, n')$ is known:

$$S_p(\omega) = \langle p^2 \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_p(\omega, n, n') \cdot \exp(i\omega n - i\omega n') \cdot dn \cdot dn'. \quad (7)$$

Further calculations of integral (7) can be accomplished by using experimental data and plausible hypotheses on the structure of the kinematic function. The results of such a semi-empirical calculation of the pressure spectrum agree with the data of direct measurements [6].

3. For hydraulic engineering practice more interesting is the case of sharply nonuniform motion. Just as in a uniform flow, the basic difficulty which impedes the theoretical calculation consists of the insufficient study of the velocity field. In connection with this, it is necessary to use the approximate representations which rest on indirect observations, comparing the results of the calculations with the data of the direct measurements of pressure fluctuations.

The calculation of pressure fluctuations on the boundary of the flow with the presence of an interface and narrow "zone of mixing" with increased eddying (Fig. 2) can be simplified, assuming the dispersion of the kinematic function nonzero only within the limits of this "zone of mixing."

The use of this condition leads to the following relationship for the space-time correlation of pressure on the bottom of the flow:

$$R_p(x, x', \tau) = \langle p(x, t)p(x', t + \tau) \rangle = \\ = \int_0^b G(x, \xi) G(x', \xi') R_f(\xi, \xi', \tau) d\xi d\xi'. \quad (8)$$

Here b is the width of the "zone of mixing," $G(x, \xi)$ - the value of the weight function in the "zone of mixing," $R_f(\xi, \xi', \tau)$ - the longitudinal time correlation of the kinematic function in the "zone of mixing."

Further calculations with the unchanged form of the correlations of the kinematic function R_f lead to different results depending on whether the weight function along the layer of mixing changes slowly or rapidly. If the zone of mixing is situated at the surface and the weight function changes relatively slowly (Fig. 2a), then the space-time correlation of the pressure fluctuations on the bottom differs significantly from the correlation of the kinematic function. Specifically, if the proper "frozen turbulence" is traced in the kinematic function, then in the pressure fluctuations on the bottom it may not appear in practice.

If, on the contrary, the weight function changes rapidly along the length of the zone of mixing (Fig. 2b), then all features of the kinematic function are brightly reflected in the properties of the pressure fluctuations on the bottom.

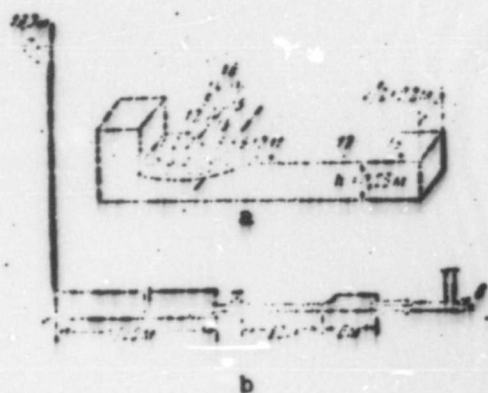


Fig. 3.

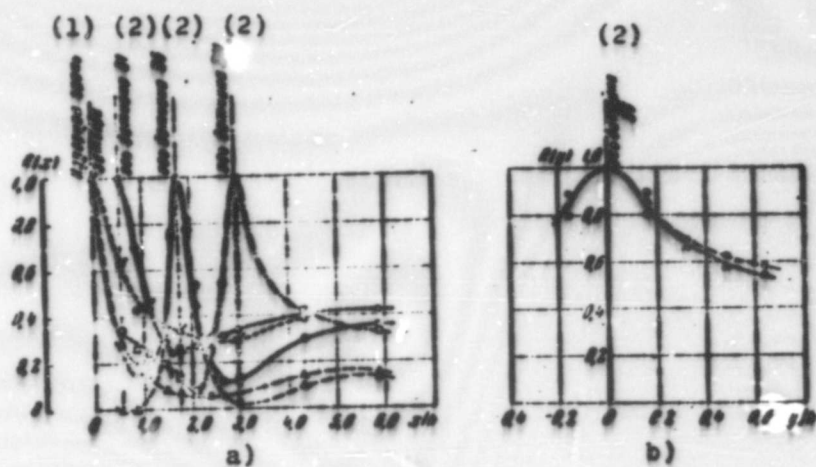


Fig. 4. The spatial correlations of fluctuations of pressure along the flow a) and across flow b).
KEY: (1) Pressure boundary of gate; (2) Axis of sensor.

This conclusion was confirmed, in particular, in experiments on a special high-pressure unit created by the Scientific Research Department of the All-Union Planning, Surveying, and Scientific Research Institute im. S. Ya. Zhuk (NIS) in the territory of the Istrinskiy hydraulic power system.

Figure 3 shows a diagram of this unit (b) and a diagram of the model of the spillway (a) on which the distribution of the correlations and standards of the pressure fluctuations in the breakaway zone was studied.

Figure 4 gives the results of the measurements of the correlations of the pressure fluctuations on the ceiling of the spillway with different locations of the base measuring point. As can be seen from Fig. 4, for the different points shown by different designations (besides triangles), a high correlation is observed at great distances between points. This means the presence, in the pressure system in question, of long-wave fluctuations not directly connected with turbulence. It was possible to suppress these fluctuations to some measure having constructed a pneumatic surge tank before the exit gate of the system. The correlation of pressure at far distances in this case was substantially reduced (triangles on Fig. 4).

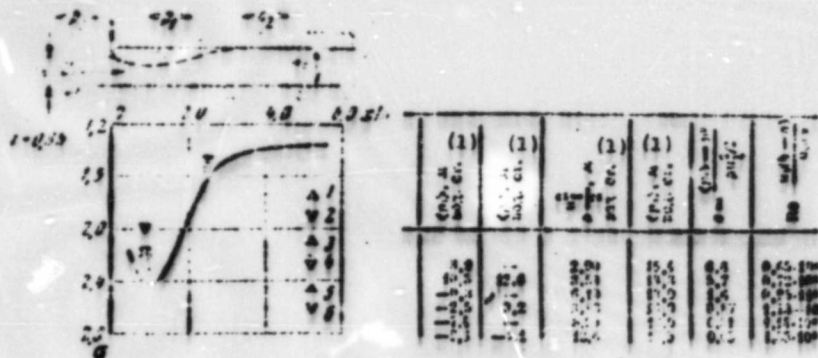


Fig. 5. Distribution of averaged pressures $\frac{(p_3 - p_1)}{L^{3/4}}$ on the ceiling of spillway in the breakaway zone.
KEY: (1) Water column.

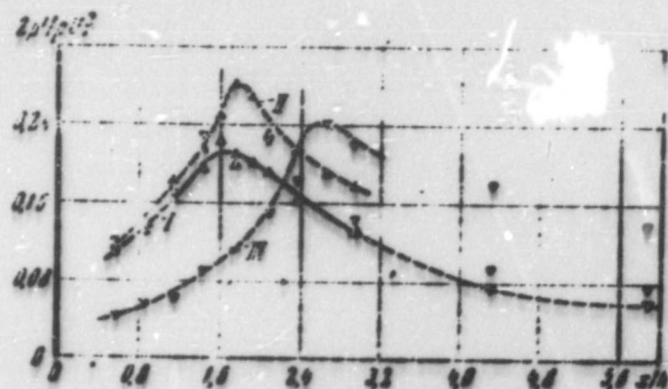


Fig. 6. The distribution of the standards of the pressure fluctuation on the ceiling of a spillway in the breakaway zone: I - in the absence of phase transitions, II - internal aeration of the flow, III - developed cavitation. The designations of the points are the same as on Fig. 5.

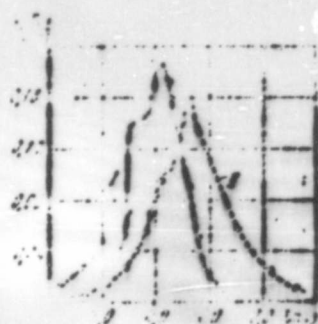


Fig. 7. The spectra of pressure fluctuations at point b (Fig. 3) in the absence of cavitation (I - $\sigma = 6.6$) and in the presence of cavitation (II - $\sigma = 0.31$).

The pressure fluctuations on the ceiling of the spillway were studied at different absolute pressures in a flow [with different cavitation numbers $\sigma = (p_0 - p_v) / \rho U^2$]. In this case it was detected that in proportion to a decrease in σ , beginning with some value of it, the standard of the pressure fluctuations first sharply increases and then falls rapidly (Figs. 5 and 6). An analogous effect was recently described by Naudascher and Locher [7]. This is explained by the qualitative change in the

structure of the fluctuations. Under conditions of the experiment with cavitation numbers less than 0.5 the intense liberation of air from the flow began (internal aeration of the flow). The amplitude of the pressure fluctuations in this case sharply increased. It was possible to note the appearance of high-frequency fluctuations directly on the oscillogram. A further reduction in the cavitation number causes a decrease in the standard of fluctuations because of the "cutting off" of declines lower than the cavitation threshold. In this case, however, the form of the spectrum of the pressure fluctuations is sharply changed - the spectrum is shifted in the direction of higher frequencies (Fig. 7). With respect to the effect on the structural elements, this change in the spectrum can be considerably more important than some decrease in the standard of fluctuations. Actually, the stresses in the plate which comprise part of the boundary of the flow and had the natural vibration frequency of about 200 Hz, with the presence of a cavitating flow, were approximately 2.5 times greater than in the absence of cavitation (with the recalculation of the experimental data on the very same dynamic pressure). Analysis of the oscillograms shows that these changes are connected mainly with the excitation of the high-frequency natural oscillations of construction (the resonant build-up of the construction).

These effects are frequently the direct cause of the failure of the facings of construction under the action of a cavitating flow.

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TURBULENT FLOWS IN JETS AND DUCTS

TURBULENT JETS

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Review

It is necessary to deal with turbulent jets of liquid, gas and a non-single phase medium in many areas of technology: aviation, rocketry, energy, metallurgical, ventilation, and others. In connection with this, in the USSR and abroad many works are being conducted which are devoted to the study of the turbulent jets and all possible jet apparatuses.

It does not appear possible to make any complete survey of the most important works on turbulent jets in one report. But there is no need for this either, since many problems of this field (jets of a noncompressible liquid, numerous applications of the theory of jets, and others) are examined in detail in monographs [1-3] as well as in well-known books and articles. Therefore, the report gives a survey of works only on turbulent jets of variable density which recently acquired an especially important significance but are still insufficiently studied.

The first work on the theory of the turbulent jet of a compressible gas was published in 1939 [4]. In this work, obtained:

from the general equations of motion, energy and continuity is the system of Reynolds equations for the turbulent flow of variable density (the terms reflecting the effect of molecular viscosity were disregarded). The transformation of equations is conducted in the usual way, whereupon each of the variable values (components of velocity, density, temperature, pressure) is replaced by the sum of its value average for time and the fluctuating addition so that in averaging for the final time interval the latter equals zero. To simplify the equations, moments of the third order are neglected and the terms which contain the derivatives of the fluctuating values along the axis parallel to the flow which are small in comparison with the derivatives of the same values along the transverse axis are disregarded. Furthermore, in accordance with Prandtl's well-known ideas, the fluctuating values are expressed by the mixing length and the gradients of average values ($u' \sim v' \sim l \partial \bar{u} / \partial y$, $\Delta T' \sim l \partial \bar{T} / \partial y$, etc.).

The obtained Reynolds equations in the general form are not integrated; therefore the analytical solution is given for the case of relatively small compressibility effect (the jet velocity is less than the speed of sound, the relative value of the difference in the temperatures in the jet and in the environment $\Delta T_0 / T_H$ is not more than 20%). In this case, the compressibility effect is considered with the use of one small parameter ($S = \Delta T_0 / T_H$), on the value of which the form of the jet boundary layer and the distribution of the velocity, temperature, and density in its cross section depends.

In 1960 a new analytical solution of the problem [1] was published, valid for a large degree of compressibility of gas; in this solution the system of equations is simplified by the use of Van Driest's method of averaging parameters in which, unlike the method described above, the density fluctuation of the current is assumed (the products of the gas density times the velocity, for example, $(\rho u)' \sim l \partial \bar{\rho} u / \partial y$; a combined value (instead of the fluctuating of velocity).

In both described methods for the calculation of the turbulent jet of a compressible gas, the equation of state for a perfect gas is used.



Fig. 1. Dimensionless profiles of excessive temperature in the cross sections of the boundary layer of cryogenic (dot-dash line), air (dotted line) and plasma (continuous line) jets (R_0 - radius of the initial cross section of the jet). KEY: (1) Calculation.

In 1961, V. A. Golubev [6] developed the theory of the turbulent jet in the case of very high gas temperature taking into account in the special form of the equation of state such factors as the dissociation and ionization of the gas; whereupon for a number of specific cases he managed to obtain the analytical solution of the system of equations. V. A. Golubev's experiments carried out with the plasma jet of water vapor flowing out into the air at temperatures of the latter up to $2 \cdot 10^4$ K confirm the theoretical solution obtained by him, which can be judged from Fig. 1, on which the experimental points and the calculated distribution curves of temperature in submerged streams of air and plasma are plotted.



Fig. 2. Curves of the critical state of Freon-22 at different temperatures. KEY: (1) Pressure, atm; (2) Molar freedom of Freon-22 in mixture with nitrogen.

In 1961, on the strength of the same positions, V. I. Bakulev developed the theory of the turbulent jet of a cryogenic substance [6, 7] which flows out into the same medium, but which remains in a gaseous state. The author selected the analytical form of the equation of state which is in good agreement with the tabular data of the thermodynamic calculation and suitable for a gas over a wide range of conditions from the temperature of liquefaction to several hundred degrees.

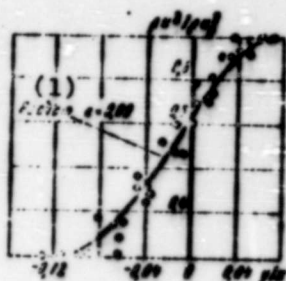


Fig. 3. Profile of the dynamic pressure in a jet of Freon-22 which flows out into gaseous nitrogen with K. A. Malinovskiy's supercritical conditions (the circle denotes experimental data). KEY: (1) Calculation.

The experimental data obtained by V. I. Bakulev, I. S. Makarov and B. G. Khudenko [7] for a jet of liquid nitrogen which flows out into a space filled with gaseous nitrogen at a temperature of 250-420°K and pressure higher than critical confirmed the theoretical calculations of V. I. Bakulev (see dot-dash curve on Fig. 1 and the corresponding experimental points). It should be noted that at supercritical pressures in these experiments, the liquid nitrogen behaved like a gas (in view of the absence of surface tension) and its mixing with the surrounding heated nitrogen bore the same character as in a single-phase medium.

In 1967, K. A. Malinovskiy [8] refined V. I. Bakulev's theory taking into account that the phase state of the substance is determined not only by pressure but also by temperature, and he constructed diagrams of the state of the mixture of nitrogen and Freon-22; such a diagram is given in Fig. 2 (the parameter for such a curve is temperature expressed in degrees centigrade). In the region lying within each given curve the substance is in a two-phased

state (wet steam), outside the curve - in a single-phase state (in the lower portion of the diagram - gas, in the upper - liquid).

K. A. Malinovskiy selected the analytical expression for the equation of state of Freon-22 similar in form to the equation used by V. I. Bakulev for nitrogen and air and conducted an experimental study of the propagation of the cryogenic jet of Freon-22 in an atmosphere of gaseous nitrogen, whereupon the basic experiments were posed with a supercritical state of Freon-22 when the mixture of this gas with gaseous nitrogen did not contain drops of Freon-22.

Using the same equations of motion and energy which were used by V. I. Bakulev and his equation of state, K. A. Malinovskiy calculated the fields of dynamic pressure in a cryogenic jet of Freon-22 which mixes with motionless gaseous nitrogen. The results of the calculation and experiment agree with each other satisfactorily, which is evidenced by Fig. 3.

It is interesting to note that a considerable increase in the width of the zone of the mixing of the initial section of the jet with an increase in the relation of densities in the external flow and in the jet ($n = \rho_H / \rho_0$), which is evident in Fig. 1 follows from theoretical calculations. It is not at all necessary to change Tollmin's constant of turbulence a which is introduced to bring experimental and theoretical profiles into conformity with the transition from a cryogenic jet to an isothermal air jet ($a = 0.09$) and only in the case of the plasma jet does it increase somewhat ($a = 0.14$); from this, it follows that in the theory of a plasma turbulent jet the compressibility effect is a little "under-considered."

In recent years, experimental and theoretical calculation work has been conducted on turbulent mixing with subsonic velocities of heterogeneous jets composed of the following pairs of gases: helium - air, carbon dioxide - air, heated air - cold air, Freon-22 - air. The effect of the relationship of velocities, densities,

temperature, and also initial conditions (degree of turbulence, relative thickness of the wall boundary layer before the beginning of mixing) on the development of the zone of mixing of the jet and of the cocurrent flow was explained. The work was carried out by O. V. Yakovlevskiy, I. P. Smirnova, A. N. Sekundov, and S. Yu. Krashenninnikov under the direction of G. N. Abramovich [9, 10]. Schlieren photographs were obtained of the jets being mixed, from which it can be seen that in general the zone of the mixing of the jets consists of three regions.

In the first, adjacent to the nozzle, the flow bears the nature of a laminar flow (with the laminar boundary layer on the nozzle walls); in the second regular vortices are formed whose size is comparable with the thickness of the zone of mixing, in which regard these vortices increase in the direction of flow; in the third, the turbulent flow regime is established (large vortices disintegrate into finer ones which move chaotically in the zone of mixing). With an increase in Reynolds number ($Re = u_0 d / \nu$, where u_0 is the velocity at the beginning of the jet, d - the diameter of the initial cross section, ν - the kinematic viscosity) the first and second regions are reduced; when $Re \approx 10^3$ the length of the wave region (in the submerged jet) exceeds three diameters of the jet, the length of the region of regular vortices comprises several jet diameters; when $Re \approx 10^4$ the length of the first region decreases to 0.25-0.5 d , and the second region - to 1.0-1.5 d .

Figure 4 presents photographs of a jet of carbon dioxide which is propagated in stationary air with two values of Reynolds numbers calculated according to the initial diameter of the jet ($Re_d = 2 \cdot 10^3$, $Re_d = 5 \cdot 10^3$); with the comparison of these photographs reduction in the wave and vortex regions is distinctly evident with an increase in the value of Re .

In the indicated work it has been established that the profiles of the dimensionless excess values of velocity, temperature, and density concentration are universal and can be expressed by the

very same curve (Fig. 5). At the same time, it was clarified that the diffusion, thermal, and dynamic zones of turbulent mixing have a different thickness. If we take as the scale of thickness the distance from the axis (or the inner boundary of the mixing zone - for the initial section) to the place in which excess velocity (or correspondingly excess temperature, or excess concentration) is half that on the axis and designate it by r_u (for velocity), r_T (for temperature) and r_k (for concentration), the relationship of these thicknesses does not depend on relationship of the velocities but changes with the relationship of densities on the axis (or on the inner boundary - for the initial section) and on the outer boundary of the mixing zone:

$$\frac{r_u}{r_T} = \frac{\rho_a}{\rho_s}, \quad \frac{r_u}{r_k} = \frac{\rho_a}{\rho_s}$$

These relations whose graphs are shown in Fig. 6, are suitable both for the main and for the initial section of the jets. In some works it is pointed out that instead of the universal distribution curve of velocity and temperature (or concentration) it is convenient to use the universal profile of dynamic pressure. The described experiments show that under conditions of a jet of variable density this hypothesis is not justified. The data of Sh. A. Yershin and L. P. Yarin definitely show that in the combustion flame (submerged jet of burning gas) the universality of the velocity profile is observed, and the dimensionless profiles of dynamic pressure with combustion and without combustion substantially differ from each other.

The same result is obtained from a study of heterogeneous jets; on Fig. 7 is plotted the profile of dynamic pressure obtained in the zone of mixing of a jet of Freon-12 with an air current with the relationship of dynamic pressures of two flows close to unity ($\rho u_r^2 / \rho u_s^2 \approx 8/7$). Nonmonotony of the curve of distribution of dynamic pressures is explained by the fact that the profile of the concentration in the transverse cross-section of the jet (and, consequently, of the density) turned out to be wider than the

velocity profile. If, from the measured profiles of dynamic pressure and density we construct the velocity profile, it proves to be monotonic and corresponds to the curves presented in Fig. 5.

The thickness of the zone of mixing of the initial section of the jet, as the experimental data show, in general depends both on the relationship of m velocities and on the relationship of n densities in the external flow and in the jet. However, if the jet and the external flow have the very same velocity ($m = 1$), the effect of n on the dimensionless thickness of the jet $b^0 = b/x$ practically ceases; the greatest compressibility effect is exhibited in the submerged jet, i.e. with $m = 0$ (Fig. 8).

The curve of the change, along the length of the jet, of the excess values of velocity and of the weight concentration (in logarithmic scale) with different relationships of velocities $m = u_H/u_0$ in the external flow and in the initial section for a jet of Freon-12 which is propagated in the airflow are depicted in Figs. 9 and 10. It is characteristic that for each of these values (Δu_m^0 and c_m) the very same picture of "attenuation" is obtained. On the basic section, the curves of drop in the corresponding value (in logarithmic scale) are practically parallel; the origin of the main section in each case can be considered the point of intersection of this curve with the line of the initial value of the corresponding quantity (for example, $\Delta \bar{u}_m = (u_m - u_H)/(u_0 - u_H) = 1$), in which regard the abscissa of this point x_n depends on values $m = u_H/u_0$ and $n = \rho_H/\rho_0$.

The experiments showed that the dimensionless abscissa of the origin of the main section $\bar{x}_n = x_n/d$ and the end of the initial section $\bar{l} = l/d$ have maximum values with identical velocities in the jet and in the cocurrent flow ($m = 1$) but decrease with an increase the density ratios in the cocurrent flow and in the jet (Fig. 11).

(1) CO_2 in the air

Fig. 4. Schlieren photograph of a jet of carbon dioxide which flows out into the air with two values of Reynolds number.

KEY: (1) Jet of CO_2 in the air. GRAPHIC NOT REPRODUCIBLE.

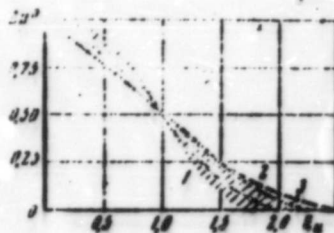


Fig. 5. Calculated profiles of the dimensionless excess values of velocities taken from different theoretical works, and the region (shaded) of the experimental values of dimensionless excess values of velocity, temperature and impurity concentration in heterogeneous jets:

- 1 - $0.5 (1 + \cos \frac{\pi}{2} \xi)$,
- 2 - $\exp(-\xi_u^2 \ln 2)$,
- 3 - $[1 - (0.44 \xi_u)^{3/2}]^2$

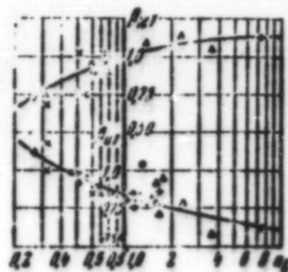


Fig. 6. The relation of ratios of thicknesses of diffusion and thermal (δ_{KT}), as well as dynamic and thermal (δ_{uT}) boundary layers of a heterogeneous jet (curves are drawn from experimental points for different relationships of velocities m in jets being mixed).

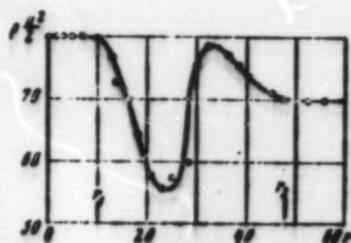


Fig. 7. Profile of dynamic pressure in the boundary layer of a jet of Freon-12 which is propagated in the cocurrent flow of air (r_1 - radius of the inner boundary of the mixing zone, r_2 - radius of the outer boundary).



Fig. 8.

Fig. 8. The dependence of the relative thickness of a jet boundary layer on the density ratio in the cocurrent flow and jet with $m = 0$ (submerged jet) and $m = 1$.



Fig. 9.

Fig. 9. Change in dimensionless excess velocity along the axis of a jet of Freon-12 which is propagated in the cocurrent flow of air with $m = \text{var}$ ($n = 0.27$).



Fig. 10. Change in the dimensionless weight concentration of Freon-22 along the axis of a jet which is propagated in the cocurrent flow of air with $m = \text{var}$ ($n = 0.27$).

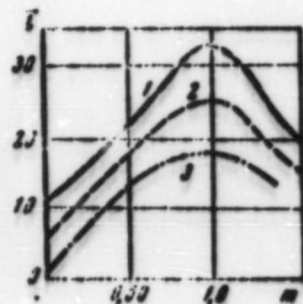


Fig. 11.

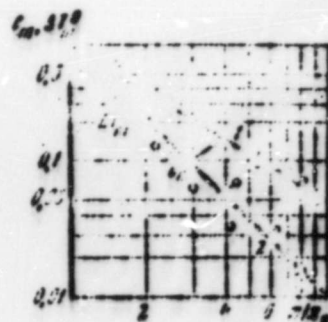


Fig. 12.

Fig. 11. Dependence of the length of the initial section of a heterogeneous jet on ratio of velocities in the cocurrent flow and in a jet with the density ratios: 1 - $n = 0.27$; 2 - $n = 1$; 3 - $n = 7.25$.

Fig. 12. Change in dimensionless values of excess temperature and gas concentration in the air along the axis of a heterogeneous jet (depending on the presented length) in two cases: the cocurrent flow is contained in a duct of constant cross section (1) or it is a submerged jet (2).



Fig. 13. Change in the degree of turbulence along the cross section of a heterogeneous jet which is propagated in a cocurrent flow in a duct (1) and in a free jet (2): ϵ - degree of turbulence, r - the current radius, R - the radius of the internal jet.

Curves of attenuation of the values:

$$\Delta \bar{u}_m = \frac{u_m - u_H}{u_0 - u_H}, \quad \Delta \bar{T}_m = \frac{T_m - T_H}{T_0 - T_H}, \quad \Delta \bar{x}_m = \frac{x_m - x_H}{x_0 - x_H}$$

along the length of the jets for different pairs of gases which constitute a jet and the external flow can be expressed with the aid of the following monomials:

$$\begin{aligned}\Delta \bar{u}_{c1} &= (x')^{-k_1}, & \Delta \bar{T}_{c1} &= (x')^{-k_1}, \\ \Delta \bar{u}_m &= (x')^{-k_1}, & \Delta \bar{T}_m &= (x')^{-k_1},\end{aligned}$$

where $x' = x/x_n$, and the exponent k_1 is a value close to unity (for weight concentration $k_c \approx 1$, for temperature $k_T \approx 1.3$; for velocity the exponent turned out to be variable in the range $0.85 \leq k_u \leq 1.25$ with an increase in n from 0.27 to 7.2). The presented values of k_1 correspond to the case of the propagation of a turbulent jet in a cocurrent flow limited by solid walls (in a tube of constant cross-section) and not subjected to preliminary artificial agitation.

A different picture is observed with the spreading of the jet within a coaxial jet of larger diameter which has a free outer boundary. In this case the high degree of turbulence in the zone of mixing of the external jet with the surrounding air is the source of the perturbations which are transmitted in a transverse direction, reaching the internal jet, and they intensify its mixing with the surrounding flow. For comparison, Fig. 12 gives attenuation curves of weight concentration c_m along the axis of the internal jet (helium in the air) and excess temperature ΔT_m^0 (air in the air), taken in two cases: the upper curve - with a cocurrent flow contained in a duct of constant cross section; lower curve - with an external flow with a free boundary (submerged jet). Figure 13 depicts distribution curves, in both cases, of the degree of turbulence across the cross section of the flow: on the lower curve corresponding to the flow in the duct, seen in the rise in the degree of turbulence in the zone of the mixing of the internal jet; on the upper curve (cocurrent flow - the submerged jet) and the degree of turbulence above and it can be seen that it is given by action from without, in which regard in the zone of mixing of the internal jet there is no "burst" of the degree of turbulence.

Monograph [1] pointed out one possible feature of the mixing of a jet with a cocurrent flow of considerably greater velocity ($u = u_m/u_0 \gg 1$), which consists in the fact that possessing a high

"ejecting capability," such a cocurrent flow intensely sucks in particles from the internal jet and, if the gas flow rate in the latter is insufficient for the "feeding" of the cocurrent flow, then directly behind that place where the internal jet "runs out," a zone of closed circulation is formed from which the "trickle feeding" of the external flow is accomplished and to which the excess of the mass then returns. Experiments confirmed the correctness of such a mixing scheme with $m \gg 1$.

As an example, Fig. 14 presents the flow lines obtained in an experiment with the mixing of an air jet with a cocurrent flow of air of the same temperature whose velocity exceeds by 27 times the jet velocity ($u_w \approx 100$ m/s, $u_0 \approx 3.7$ m/s). The boundary of the circulation zone is depicted by a continuous line; the boundary of the region of reverse current is shown by the dotted line; plotted on Fig. 14 are the experimental points from which the flow lines are drawn.

The described special case of the formation of the internal separation of the flow (not from the wall but with the presence of a tangential velocity discontinuity), but already with supersonic speed, was also encountered by American researchers [11], who work under the guidance of A. Perry, and about which the latter reported in the USSR in the spring of 1966.

The turbulent jet which is extended in a motionless medium (submerged jet), captures (ejects) the particles of this medium and because of this excites the general relatively slow movement of the liquid (irrotational flow) toward its boundaries.

In the works of L. D. Landau and Ye. Lifschitz [12], V. V. Pavlovskiy [13], O. V. Yakovlevskiy and A. I. Lekundov [14] and A. S. Ginevskiy [15] theoretical and experimental studies were conducted of the irrotational flow of a submerged jet in the case of a flat axially symmetrical jet and with different positions of the outer boundaries of the medium embracing the jet.

The theoretical solution of the problem of external irrotational flow is obtained according to the distribution of the transverse velocity component on the jet boundary known from calculation. Three examples of irrotational flow outside the jet are illustrated by Figs. 15-17 taken from work [14]. Figure 15 gives the flow line in the case of a plane jet which flows out from a slot in the wall perpendicular to the jet direction; a characteristic feature of this flow is the fact that the direction of its flow lines at the jet boundary is opposite to the jet direction.

Figure 16 presents the flow lines for an analogous case of the outflow of an axially symmetrical jet. Figure 17 depicts the flow lines outside the axially symmetrical jet flowing out from a nozzle into unlimited space (near the nozzle exit there are no enclosing walls). The results of the theoretical calculations and visual-quantitative experiments (outside the jet streams of smoke are photographed which are arranged along the flow lines) agree well with each other. Knowledge of the flow which arises outside the jet is very essential to evaluate the aerodynamic forces which act on the bodies arranged beyond the limits of the jet. For example, in this way it is possible to determine the supplementary aerodynamic force which acts on a jet airplane during vertical takeoff (the force is caused by the intense sucking-in of air to the jet stream which spreads over the surface of earth - Fig. 18).

In a number of technical devices (furnaces of boiler units, the combustion chamber of gas-turbine engines, vertical takeoff jet aircraft moving near the surface of the earth, ventilation air screens, etc.) it is necessary to deal with a turbulent jet being blown off by a lateral flow. The axis of such a jet is distorted, the boundaries of a mixing zone are asymmetric relative to the axis, and the law of velocity change along the axis differs significantly from an analogous law for the case of a jet with a straight-line axis.